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Students' Hypothetical Learning Trajectory (HLT) in Learning Fraction Division Calculation Operations

Rosmayasari[1*], Didi Suryadi[2], Tatang Herman[3], Sufyani Prabawanto[4], Toh Tin Lam[5]

[1*]School of Postgraduate Studies, Universitas Pendidikan Indonesia, INDONESIA
[2, 3, 4] Departement of Matematics Education, Universitas Pendidikan Indonesia, INDONESIA
[5]Mathematics & Mathematics Education Academic Group, National Institute of Education Singapore, SINGAPORE

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ABSTRACT

This research aims to obtain a comprehensive picture of the hypothetical learning trajectory (HLT) that should be developed based on students' learning trajectory (LT) in learning fraction division arithmetic operations in grade V elementary school. The HLT was developed based on the analysis of the Learning Implementation Plan document made by the teacher by considering the students' learning trajectory on the material of fraction division arithmetic operation, including the learning carried out by the teacher in the classroom, examining aspects of learning obstacles that occur in the learning process, and examining what didactical situations will be built, predicting student responses that may occur to the situation created, and determining didactical and pedagogical anticipation of these responses. The HLT in this study started with the context of generating the idea of number division, learning the concept of number division such as natural numbers divided by natural numbers, and recalling the concept of fractions and the concept of division of fractions. The learning objective designed in the HLT is that students can solve problems with at least two ways related to division of fractions.

Keywords: Hypothetical Learning Trajectory, learning obstacle, fraction division calculation operation

INTRODUCTION

Fractions are one of the most complex and important mathematics materials in elementary school [1], [2]. Learning mathematics on fraction material is still considered difficult [3], [4], [5], [6]. The difficulty in learning the concept of fractions is because the ability to understand fractions as part of a whole expressed in symbols requires special understanding. The concept of fractions is seen as a concept that is difficult to learn and difficult to teach. This can pose pedagogical challenges for teachers on an ongoing basis, especially in teaching mathematics [7]. One of the main factors contributing to the complexity of learning fractions is the multi-faceted notion that includes five interrelated sub-constructions (i.e., part-whole, ratio, operation, quotient, and measure) [8]. Much of the confusion with fractions are related to different interpretations (constructions), representations (models), and coding conventions. The debate causes fraction material to be considered difficult by students [9]. Fractions are a topic area that many teachers are challenged to learn and teach [10].

Fraction material that is considered difficult is fraction calculation operations. In the fraction calculation operation material, students' difficulties are when solving story problems. The cause of student difficulties is due to students' lack of understanding of the basic concepts of fractions, misapplication of fractions in solving fraction problems,

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✉ rosmayasari_alby@upi.edu **(Correspondence)**

carelessness in understanding the language of the problem, lack of understanding of the prerequisite material, and errors in the computation or calculation process [11], [12], [8], [2]. Student errors related to story problems, incorrect generalization of learned rules for fractions, considering numerators and denominators as whole numbers, not learning the division operation of fractions conceptually, and associating division with addition, subtraction, and multiplication operations [13]. The causes of errors made by students in solving fraction arithmetic operations include students lacking understanding of the concepts of addition, subtraction, multiplication, and division of fraction arithmetic operations, students lacking ability in systematic steps to solve fraction arithmetic operations, and students lacking accuracy in performing calculations [13]. [14].

The material for calculating the division of fractions, when viewed from the KI and KD in Permendikbud-RI No. 37 of 2018 [15], is found in grade V with KD "3.2. Explain and perform multiplication and division of fractions and decimals and 4.2. Solve problems related to multiplication and division of fractions and decimals". Furthermore, to further strengthen the initial suspicion related to problems in the material of calculating the division of fractions, the researcher conducted a preliminary study. The study was conducted on mathematics learning in grade VI public elementary schools in Bandung City, especially on the material of fraction division calculation operations in the form of story problems. The reason for choosing grade VI as a preliminary study was because the position at the time of data collection was carried out at the beginning of the odd semester and students had already received material on the calculation of division of fractions in the previous class.

The results of observations and interviews with teachers found that learning activities had not been directed at a problem and ended in problem solving. The nature of good mathematics learning is that it begins with the submission of problems and aims to solve problems [16]. This is what seems to have not been well developed in the student learning process on the material of calculating the division of fractions. As previously stated, the learning interaction is still dominated by the teacher when discussing material and solving a problem. Students are less involved in the process.

An indication of the lack of or perhaps not going through a series of mental actions students can cause students' way of thinking related to fraction operation material to be limited [16]. The limited way of thinking of students related to fraction arithmetic operation material also results in how to generate understanding and how to find problem-solving strategies in students related to fraction arithmetic operation material is not well facilitated. So, it is necessary to design learning activities that can create situations where students can develop their understanding of the problems given.

The services provided by teachers in helping to overcome students' learning obstacles in the material of calculating the division of fractions are not optimal. The activities carried out by the teacher are only question-and-answer activities and re-explanation of learning materials without finding out the causes and solutions to overcome these obstacles. So that the learning objectives that have been set are not achieved. Students should not be treated as passive recipients who only receive material by using formulas and procedures to solve a problem. Students should be given the opportunity and guided to situations to rediscover mathematical concepts in their way [17].

These findings become the basis for researchers to design learning activities based on students' learning trajectories. To facilitate the differences in students' learning trajectories, teachers need to design a hypothetical learning trajectory (HLT) so that the designed

learning objectives can be achieved. This research aims to obtain a comprehensive picture of the hypothetical learning trajectory (HLT) that should be developed based on n student's learning trajectory (LT) in learning fraction division arithmetic in grade V elementary school.

MATERIAL AND METHODS

HLT is designed based on the learning objectives to be achieved, activities that support the objectives, and mathematical hypotheses in the form of conjectures that are expected to occur to students according to their thinking abilities [18]. The HLT was prepared based on the results of the analysis of the Learning Implementation Plan (RPP) document made by the teacher by considering the student's learning trajectory on the material of fraction division calculation operation. The steps taken by the researcher in preparing the HLT were as follows: 1). Theoretical study of students' learning trajectory in the age range of 10 - 11 years (grade V); 2). Studying the history and in-depth study of the theory of fraction division calculation material; 3). Review the curriculum and mathematics textbooks used by grade V students, including the learning conducted by teachers in the classroom; 4). Examine aspects of learning obstacles that occur in the learning process and minimize learning obstacles from both student and teacher aspects; and 5). Examine what didactical situations will be built, predict students' responses that may occur to the situations created, and determine didactical and pedagogical anticipations of these responses.

RESULTS AND DISCUSSION

The results of the analysis of the Learning Implementation Plan (RPP) document made by the teacher found a mismatch between the material, method, and use of media and learning design with the demands of thinking and student characteristics. This mismatch causes didactical barriers for students. Didactic barriers as learning obstacles or difficulties caused by the state of the didactic design used or the teacher's didactic intervention [19]. Some things that need to be considered in designing lesson plans and implementing learning such as the ability to compile and present material must pay attention to the order of the material, both structurally (interrelationships between concepts), as well as functionally (suitability for students' level of thinking), and the stages of presenting the material. Selection of methods that can make students active in learning and explore the potential of students' thinking in solving problems. The use of media can facilitate students' understanding of the material. The function of teaching aids is to avoid the abstractness of a concept and can capture the true meaning of the concept [20]. For this reason, in choosing objects around students that will be used as props in instilling an understanding of the concept of division of fractions must be careful. If the selection of media is not appropriate, it is likely, the concept of division of fractions that will be instilled will not be captured properly by students.

Three reasons for the low conceptual understanding of students on fractions from The Trends in International Mathematics and Science Study (TIMSS) results are 1) the content of the Indonesian curriculum which places low emphasis on the basic concept of

fractions and introduces fraction operations too early; 2) Indonesian mathematics textbooks present only one definition of fractions, namely as part of a whole; and 3) there is limited use of models or representations of fractions in classroom practice [21].

Problems in fractions need to be discussed and found a solution because fraction material in the 2013 curriculum is not only taught in elementary schools starting from grade II to grade V and in junior high schools (SMP) is also taught in grade VII with a lot of material coverage [22]. In addition, teachers must also know the juridical basis underlying the importance of learning fractions, by reviewing the Core Competencies (KI) and Basic Competencies (KD) of the lessons in the 2013 Curriculum, especially the content of mathematics subjects in elementary school as stated in the Regulation of the Minister of Education and Culture of the Republic of Indonesia No. 37 of 2018 [15]. Then, making a comparison of the curriculum in Indonesia with the curriculum in the National Council of Teachers of Mathematics (NCTM) and other developed countries such as Singapore, Japan, and Finland, as shown in Figure 1 below:

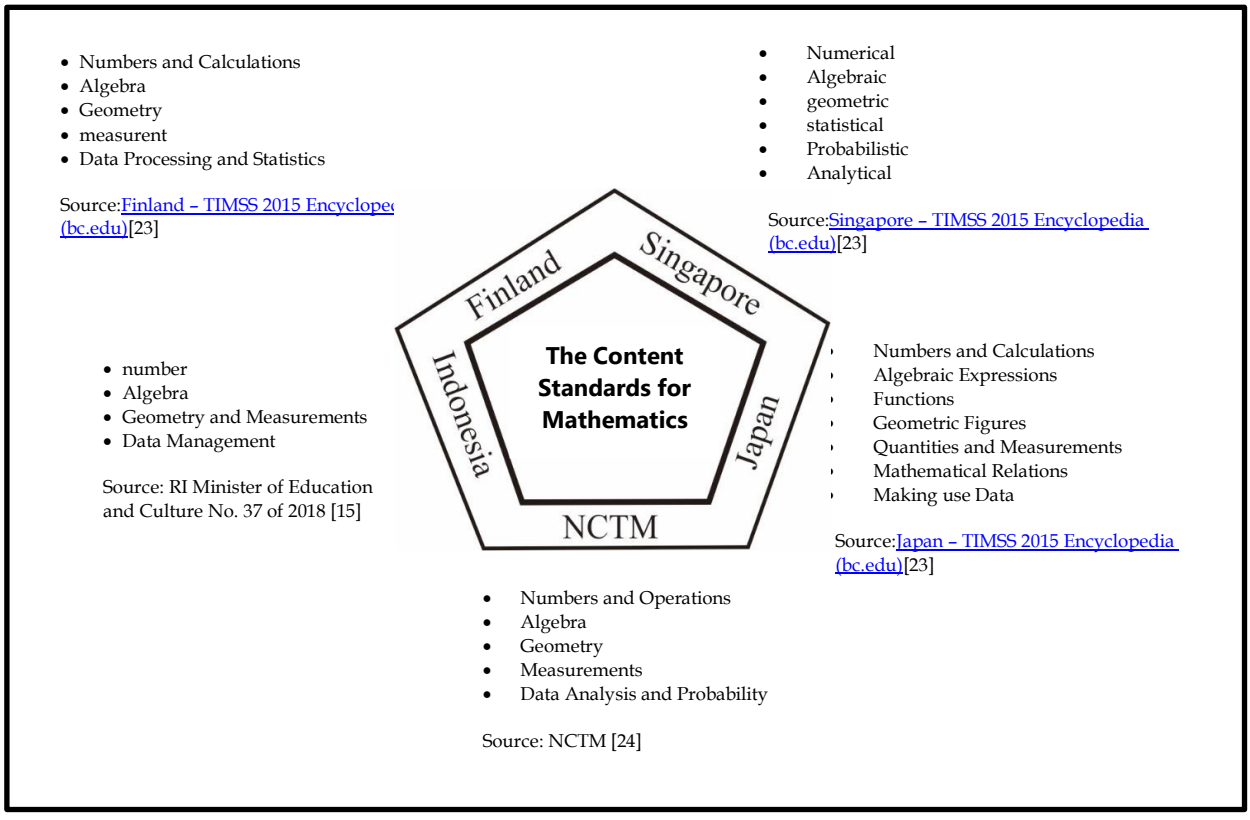

Figure 1. The Content Standards for Mathematics

Figure 1 above, explains in general the comparison results obtained that mathematics learning materials include Numbers, Algebra, Geometry, and Statistics. Number material in NCTM and developed countries is given at the beginning of learning mathematics as well as in Indonesia. This is not without reason, because number material is a prerequisite for students to learn the next material. The reasons why numbers are so important for students to learn, such as: (1) number is the first material taught in school formally; (2) number is a basic part of the material in mathematics; (3) number operations and applications are related

to real-life situations that exist in it [25].

Creating a classroom environment that normalizes errors as part of the learning process is something that teachers need to do. It is important to involve students in assessing mathematical errors and misconceptions. This is to ensure that students have a deep conceptual understanding, as well as to link new knowledge to prior knowledge correctly [26]. Misconceptions can never be completely avoided, but teachers can intervene before misconceptions take root. First, teachers must understand why their students make mistakes or how they develop misconceptions before they can address them and develop interventions to promote correct understanding [27], [26]. What teachers can do is anticipate the learning barriers that students experience in learning. Students need to be directed in their way of thinking in understanding knowledge in learning mathematics. In addition, teachers also need to pay attention to students' learning flow.

Hypothetical Learning Trajectory (HLT) contains a series of instructional tasks to provide students with an understanding of mathematics learning concepts. HLT is one of the important aspects that must be designed by teachers to make learning more meaningful [28]. In this study, HLT was developed based on the findings of learning obstacles experienced by students, causal factors, and learning objectives that were not achieved during the learning process of fraction division calculation operation material before the application of the initial didactical design. The learning objectives designed and not achieved were as follows: 1). Through the activity of manipulating folding paper, students can explain the concept of division of fractions correctly; and 2). Through the teacher's explanation, students can solve fraction division arithmetic operation problems correctly (Source: teacher's lesson plan). These learning objectives are developed based on Basic Competency (KD) 3.2. Explain and perform multiplication and division of fractions and decimals and 4.2. Solve problems related to multiplication and division of fractions and decimals (Source: Teacher's lesson plan).

The preparation of HLT according to Simon [29] explained that the presumptive learning path is an activity plan prepared by the teacher in anticipating possible student learning paths by considering knowledge acquisition, level of understanding, and selection of learning activities with mathematics learning objectives. The preparation of the presumptive learning path is based on the learning objectives to be achieved in the form of learning stages in the form of a series of didactical situations that are mutually sustainable [30], [31]. The preparation of HLT needs to pay attention to the stage of thinking and cognitive development of students [32]. Gravemeijer [33], [32] described the levels of conceptual learning trajectories, namely: situational, referential, general, and formal. At the situational level, the student's position is in the context of a particular situation. Referential level, the explanation of the situation in the problem refers to the application of models and strategies. The general level is on the mastery of strategies based on the given context. Formal level, performing actions by applying conventional procedures and notations. After the four stages are passed by students, they are expected to be able to apply the concept to new problems in different contexts.

Based on the review and analysis of the researcher's findings, the HLT designed for learning fraction division calculation materials is presented in Figure 2.

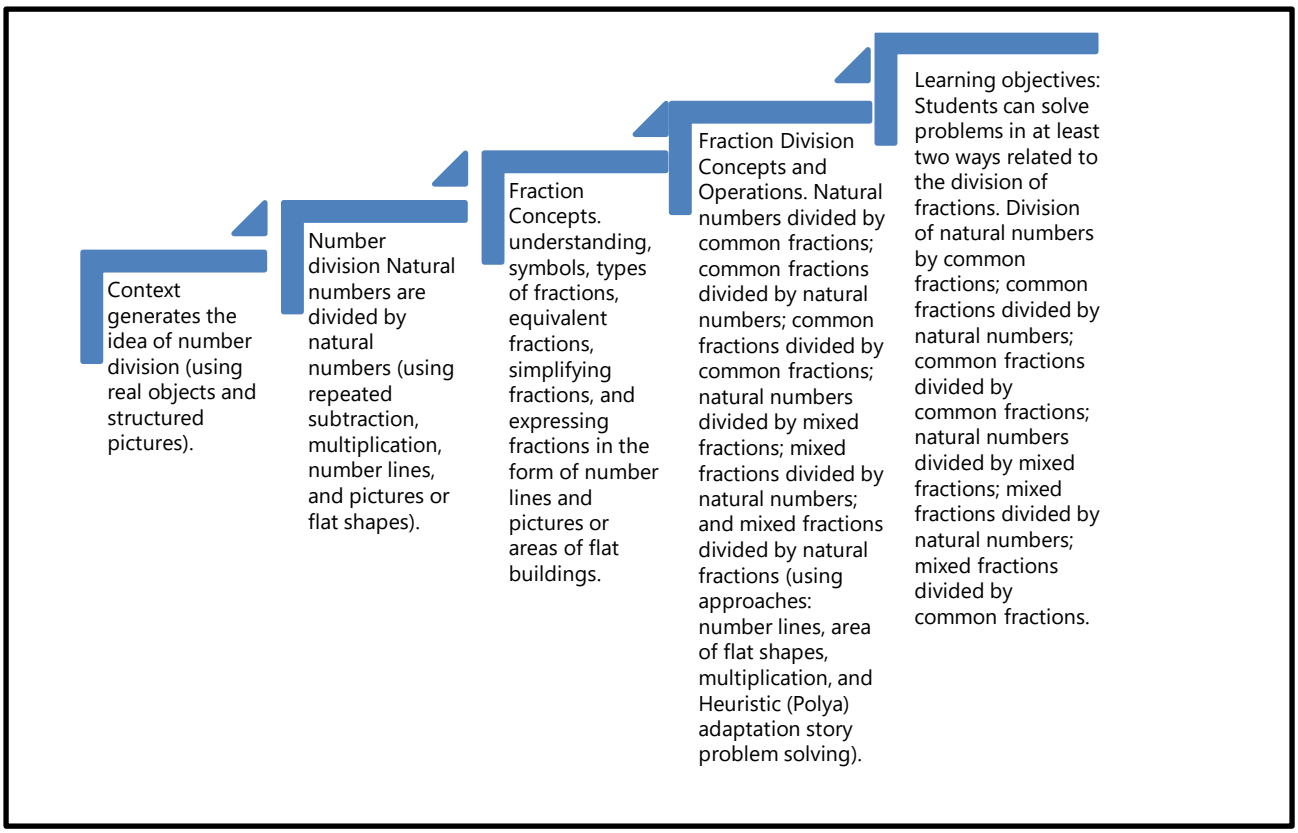

Figure 2. HLT for learning the division of fractions

Figure 2. explains that HLT in learning division of fractions starts with the context of bringing up the idea of number division such as students being given problems that are close to students' daily lives. The problem-solving process uses the help or manipulation of real objects and structured images. Furthermore, the learning of number division concepts such as natural numbers divided by natural numbers whose solutions use repeated subtraction, number lines, pictures, or flat shapes without using algorithms first, then algorithms.

The next lesson is to recall the concept of fractions such as understanding, symbols, types of fractions, fractions worth, and simplifying fractions. In the learning process, students can use real objects, folded paper, and paper with patterns, that Students are expected to be able to express fractions in the form of number lines, drawing approaches, or the area of flat buildings with square, rectangle, triangle, circle, trapezoid, and parallelogram shapes. After students understand the concept of fractions, they proceed with the concept of division of fractions, such as students are introduced to the definition of division of fractions, the application of division operations on types of fractions, and how to solve division on fractions such as using the number line approach, pictures or flat area

approach, multiplication approach and problem-solving story problems adaptation of Polya's Heuristics [34].

The learning objective designed in the HLT is that students can solve problems in at least two ways related to the division of fractions. The form of problem is packaged in the form of story problems and mathematical sentences. The two ways in question are adjusted to the form of the problem given, for example, the problem given in the form of a mathematical sentence can use a number line and a picture or a flat area approach, a number line and a multiplication approach, a picture or flat area approach, and a multiplication approach. If the problem given is a story problem, then the way to answer it can use a number line and a picture or a flat area approach, a number line and a multiplication approach, a number line and solving Polya's Heuristic story problem, a picture or a flat area approach and a multiplication approach, a picture or a flat area approach and solving Polya's Heuristic story problem, a multiplication approach and solving Polya's Heuristic story problem.

The division calculation operations on fractions introduced to students are a division of natural numbers by natural fractions; natural fractions divided by natural numbers; natural fractions divided by natural fractions; natural numbers divided by mixed fractions; mixed fractions divided by natural numbers; mixed fractions divided by natural fractions, and mixed fractions divided by mixed fractions.

Based on the material description above, the researcher made a concept map related to the division of fractions presented in the form of a chart. This aims to provide an overview of the limitations or breadth of the material to be studied. In the chart, it is briefly studied starting from the definition of division of fractions, namely the division operation that applies to fractions, the application of division operations on types of fractions, and how to solve the problem. Figure 3. below, presents a chart of fraction division operation material.

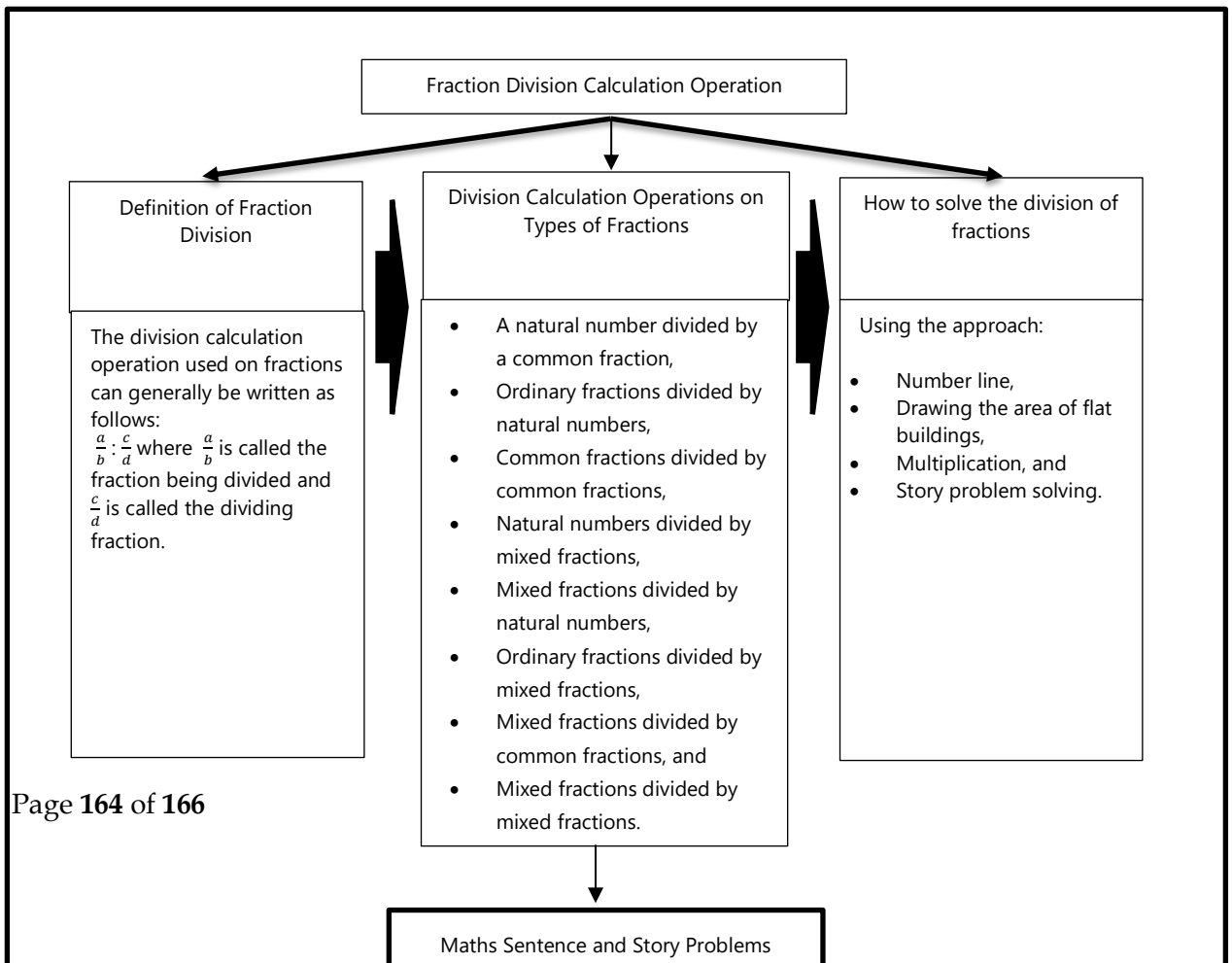

Figure 3. Concept of fraction division calculation operation

Figure 3 explains that to achieve the learning objectives, students first understand the position of the fraction or number being divided and the fraction or number divider, students are also allowed to learn the application of division operations on types of fractions to make it easier to search and find various alternative ways to solve problems related to the division of fractions.

CONCLUSIONS

The Hypothetical Learning Trajectory (HLT) of students in learning fraction division arithmetic operation material in grade V SD starts with the context of bringing up the idea of number division such as students are given problems related to students daily lives. As for the problem-solving process using the help of or manipulating real objects and structured images. Furthermore, learning the concept of number division such as natural numbers divided by natural numbers whose solutions use repeated subtraction, number lines, pictures, or flat shapes without using algorithms first, then algorithms. The next learning is reminding the concept of fractions such as understanding, symbols, types of fractions, fractions worth, and simplifying fractions. In the learning process, students can use real objects, folded paper, and paper with patterns, that Students are expected to be able to express fractions in the form of number lines, pictures, or approaches to the area of flat buildings of various shapes such as squares, rectangles, circles, triangles, trapezoids, and parallelogram. After students understand the concept of fractions, they proceed with the concept of division of fractions, such as students are introduced to the definition of division of fractions, the application of division operations on types of fractions, and how to solve division on fractions using the number line approach, pictures or flat area approach, multiplication approach and problem-solving of story problems adapting Polya's Heuristics. The learning objectives designed in the HLT are that students can solve problems in at least two ways related to the division of fractions.

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