

On Some Guiding Principles of Enacting Mathematical Problem Solving for Classroom Instruction

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ABSTRACT

In addressing the key role that problem solving has been playing in mathematics instruction for K-12, this paper aims to assist mathematics teachers and educators to consider a set of guiding principles for designing problem solving tasks for classroom instructions. The set of guiding principles was synthesized and proposed through the researchers' systematic review of existing education literature on problem solving.

Keywords: Mathematics Education, Mathematics Instruction, Mathematical Problem Solving, Teaching Through Problem Solving, Teaching For Problem Solving

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For the last few decades, problem solving has been the central focus of many mathematics curricula globally. In the United States, for example, problem solving has been recommended by the National Council of Teachers of Mathematics (NCTM) as a central focus in the mathematics curriculum since the 1980s [1]. In the United Kingdom, the national curriculum for mathematics introduced in 2014 aims to ensure that all students "can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions" [2, p.4]. In Australia, leading mathematics education groups put out a joint statement pushing for students to learn more problem-solving skills stating that "we must ensure that there is an ongoing commitment from all stakeholders to deliver effective professional development that gives our teachers the skills to teach not just the content but the skills and competencies necessary" [3].

However, the enactment of problem solving in mathematics classrooms has generally not been widespread in the mathematics classrooms. According to Wilkerson, "[teachers and educators] still have been challenged to fully embrace problem solving at times, often finding a continued focus on learning procedures, a lowering of expectations for some students, and limited access to instructional resources to support problem solving" [4]. A recent report by Ofsted highlighted that problem-solving in classrooms are being hindered by the fact that "pupils need to be fluent with the relevant facts and methods before being expected to learn how to apply them to problem-solving conditions" [5, p. 17].

In Singapore, teachers are also facing difficulties in enacting problem solving. The Singapore education system is largely centralized with most schools following the common curriculum proposed by the Singapore Ministry of Education (MOE). The majority of secondary school students in Singapore attend government schools under the GCE O-Level Programme, where Mathematics is a compulsory subject and taught in

accordance with the core curriculum [6]. Developing students' ability in problem solving have been part of the mathematics learning objectives since the 1970s [7]. In the 1990s, mathematical problem solving was made a central part of Singapore's secondary school mathematics curriculum [8]. Despite various revisions to the curriculum over the decades, mathematical problem solving still remains at the centre of mathematics curriculum framework up to the latest 2020 secondary school syllabus seen in **Figure 1** [9] below.



Figure 1. Singapore mathematics curriculum framework.

The Singapore MOE syllabus also defines problems to include "complex and nonroutine tasks that requires deeper insights, logical reasoning and creative thinking" [9, p. 9]. General problem solving strategies, commonly known as heuristics, were proposed as plausible approaches to tackling a problem. Polya's four-step problem solving model was recommended in the curriculum document for students to handle non-routine problems.

Despite the long history of mathematical problem solving in the Singapore mathematics curriculum, up to the last decade teachers still struggle with the dichotomy between developing fluent basic skills and problem solving ability in the classroom [10]. The research projects Mathematical Problem Solving for Everyone (MProSE) and MProSE: Infusion and Diffusion (MInD) were introduced by a group of researchers from the Singapore National Institute of Education to infuse mathematical problem solving into the mathematics classrooms of secondary schools [11]. The researchers curated a mathematical problem solving module based on the work of Polya and Schoenfeld. Classroom teachers can use the module when teaching about problem solving [12]. However, the project faced several challenges when attempting to make problem solving a regular feature in mathematics classrooms:

i) Singapore secondary mathematics teachers often make effort to modify challenging tasks so that students can engage with them cognitively and affectively [13]. The problems in the MProSE modules were adapted by teachers to suit the local conditions of each school. However, as teachers who participated in the research projects did not design the original problems, they lacked insight into the mathematical and theoretical considerations which underlie the choice of the problems. "In their attempts at the local level to chop, add, replace, and re-order, teachers may disrupt the balance among

the various components and distort the intended problem solving developmental trajectory embedded in the MProSE lessons." [14, p.110].

ii) "Another challenge to teaching problem solving is the lock-step grid of fixed teaching schedules." [15, p. 319] Teachers in Singapore are expected to cover the syllabus content in a timely manner and help their students exceed in examinations [16]. Hence, allocating additional curriculum time to problem solving could be unrealistic for most teachers. When modifications were made to move the MProSE module from outside-curriculum to within-curriculum, "[the] teachers were very conscious of class time taking up problem solving" [15, p. 318]. The time consumption of the problems contributed to the resistance towards the infusion of problem solving in classrooms.

In this paper we propose a set of guiding principles that can be used by educators when designing problem solving tasks to be enacted in a mathematics classroom. The design principles are for teachers to take into consideration when creating or modifying a problem solving task to ensure that the affordances of the task are not lost in the process. Furthermore, the design principles focus on teaching for/through problem solving instead of teaching about problem solving to address the issue of the perceived lack of curriculum time for problem solving.

MATERIAL AND METHODS

This section outlines the methodology employed in this research to derive a set of guiding principles for designing mathematical problem-solving tasks through an extensive literature review. The methodology comprises the following key stages: (1) literature selection and review, and (2) synthesis of guiding principles.

The initial phase of this research involved a systematic literature search to identify pertinent studies, articles, and documents relevant to the research topic. Databases such as ERIC, Scopus, Google Scholar, and relevant academic journals were utilized to conduct the search with keywords and phrases aligned with the scope of the research. Inclusion criteria for the literature encompassed peer-reviewed articles, books, reports, and established frameworks that presented concepts, models, or strategies related to the subject area. To ensure a comprehensive understanding of the topic, we took into consideration both seminal and contemporary works.

The guiding principles proposed in this research were derived through an iterative process of synthesizing the concepts, ideas, and methodologies presented in the selected literature. The literature was critically analyzed to identify recurring themes, patterns, and core recommendations that could serve as the foundation for the guiding principles. To facilitate the synthesis, a thematic analysis approach was employed. The selected literature was organized into categories based on thematic similarities. Within each category, commonalities and variations were identified, and potential guiding principles were formulated accordingly. The emerging set of principles was then cross-referenced with the literature to validate their alignment and representation of the reviewed material.

RESULTS AND DISCUSSION

Mathematical Tasks Framework

A mathematical task is a set of instructions given to students to initiate mathematical activity, which is the engagement students have with a mathematical idea [17]. Teachers need to consider the mathematical activities afforded by the tasks during the design and implementation of the tasks in class.

In this study, we studied Stein et al's Mathematical Tasks Framework used to investigate the relationship between teacher instruction and student learning. The key stages through which teachers' treatment of mathematical tasks can impact student learning include (1) setting up of the task by teacher; (2) implementation of the task by students.

The first stage is the setting up of the mathematical task, which refers to how the task as represented in instructional materials is being presented by the teacher to the class. Factors such as the teacher's goals, content knowledge, and knowledge of students could impact this stage. The focus of the investigation by Stein et al. was on the link between the setting up of a mathematical task and its implementation, which refers to the mathematical activity carried out by the students. The link between mathematical tasks and the corresponding mathematical activity were examined in terms of task features and cognitive demands.

Stein et al. defines task features as the "aspects of tasks that mathematics educators have identified as important considerations for the engagement of student thinking, reasoning, and sense-making" (e.g. the existence of multiple-solution strategies, the extent to which the task lends itself to multiple representations, and the extent to which the task demands explanations and/or justifications from the students), while cognitive demands refer to "the kind of thinking processes entailed in solving the task" (e.g. memorization, the use of procedures and algorithm, and the employment of complex thinking and reasoning strategies). The factors that affected implementation were proposed to be classroom norms, task conditions, teacher instructional habits and dispositions, and student learning habits and dispositions. Further., the factors associated with the decline and maintenance of high-level cognitive demands from the set-up phase to the implementation phase are (1) challenge becomes nonproblem, (2) inappropriateness of task for students, (3) focus shifts from the processes to correct answer, (4) inappropriate amount of time, (5) lack of accountability, (6) classroom management problems, among other problems.

Problem Solving

A problem is a task that is non-routine for which the solution is not immediately forthcoming [19]. Further, an individual or group must be motivated to find a solution and attempts to do so for the task to be considered a problem [20]. Schoenfeld believes that "the primary responsibility of mathematics faculty is to teach their students to think: to question and probe, to get to the mathematical heart of the matter, to be able to employ ideas rather than to regurgitate them" [21, p. 2] and recommends that problem solving lessons be integrated in standard mathematics curriculum in order to achieve that goal.

Polya's well-known problem-solving model consists of four steps: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back [19]. Schoenfeld, building on Polya's four-step problem solving model, proposes that there are four aspects of mathematical thinking that can contribute to problem solving behaviour: resources, heuristics, control, and belief systems [22].

Teaching about, for, and through Problem Solving

Problem solving can be enacted in a mathematics classroom in various ways. Schroeder and Lester introduced three approaches to problem-solving instruction: teaching about, for, and through problem solving [23]. These conceptions still remain useful as descriptions of enactments of mathematical problem solving in classrooms [15].

Teaching about problem solving uses problems to teach students problem solving skills (e.g., Polya's four-step model, heuristics, problem solving disposition) that they can use to solve problems. An example of a collection of resources in this area can be found in the book Making Mathematics Practical [12]. Most secondary school mathematics classrooms currently adopt the instructional approach of teaching for problem solving. In teaching for problem solving, students are taught certain mathematical knowledge before they are posed problems which solution requires the mathematical knowledge taught [24]. On the other hand, teaching through problem solving is used to describe instructions in which teachers engage their students in problem solving processes with the acquisition of mathematical content knowledge as the end goal [25].

The distinctions between teaching about, for, and through problem solving is important if teachers are to make use of problem-solving tasks in the classroom as it impacts which stage the problem-solving task should be introduced during a lesson [23].

Concepts and Skills

The use of appropriate mathematical concepts and skills is one of the key elements that distinguishes teaching for/through problem solving from teaching about problem solving. Mathematical concepts and skills make up two sides of the Singapore mathematical curriculum framework as seen in **Figure 1** [9] above. The teacher should first determine the learning objectives associated with a given task, which can be based on the syllabus content [26]. The mathematical concepts and skills that are needed to solve the designed problem and its possible extensions should align with the mathematical ideas in the learning objectives. Although the designed problem might lend itself to the examination of mathematical ideas outside of scope of the learning objectives, it is crucial that the problem is at least mathematically rich enough for the learning objectives to be fulfilled.

Instructional Objectives

Once the learning objectives have been identified, the teacher needs to determine the instructional objectives. "Learning objectives (learning outcomes) are targets for student learning, while instructional objectives (specific instructional objectives, or SIOs) are the purposes of teacher teaching." [27, p. 247]. Instructional objectives must be observable and/or measurable [28]. They can be described using action verbs found in Bloom's revised taxonomy [29] which covers the cognitive, affective, and psychomotor domains [30]. At this stage, teachers decide their instructional approach, that is, teaching for or through problem solving. This will likely impact the setting of instructional objectives.

The instructional objectives when teaching for problem solving would be for students to apply the mathematical content and skills in the learning objectives in solving the problem. However, the instructional objective when teaching through problem solving would be for students to explore mathematical content and skills in the learning objectives in the process of solving the problem. Furthermore, the instructional objectives can include mathematical processes as seen in **Figure 1** [9] above. It noteworthy that the same problem could be used for either teaching for/through problem solving, with the difference being the instructional approach.

Learners' Schema

The schema is the basic unit necessary for mental organization and mental functioning [31]. It is a set of knowledge about a concept, that can be applied to any instance of that concept [32]. Accommodation modifies an existing schema to understand new information that cannot be understood using the existing schemata (plural of schema). When new encounters disrupt the balance between them and their environment, accommodation restores the equilibrium. The designed problem should create a state of disequilibrium in the students, thus motivating them to activate prior schemata and undergo accommodation [33].

It is therefore important to consider the schema students require to solve the problem as well as extensions of the problem. Teaching for problem solving would require students to understand the mathematical ideas needed to solve the problem before attempting it. On the other hand, problems designed for teaching through problem solving should allow students to learn new mathematical ideas through solving the problem and/or its extensions. The designed problem should make use of students' prior knowledge and the process of problem solving should allow students to proceed through their "zone of proximal development" [34] and lead them to new mathematical ideas.

Teachers scaffold "students' ideas so as allow them to extend and move forward as well as initiating, focusing and highlighting new mathematical ideas and thinking, and important mathematical practices" [35, p. 137]. Furthermore, Hill et al. claimed that teachers require a firm grasp of subject knowledge to develop students' mathematical understanding [36]. Hence, teacher should try solving the problem and its extensions using multiple methods to uncover the mathematical richness of the problem [21]. This allows the teacher to be sufficiently prepared to investigate the relevant mathematical ideas with the class and guide them towards the intended learning objectives if necessary.

Gathering Information and Providing Feedback: Assessment

Assessments gather information and provide feedback to support student learning and improve teaching practice [37]. Brookhart and McMillan revises the definition of classroom assessments to include that "[they] may be designed by the teacher or may be externally designed and selected by the teacher ... However, they must be locally controlled by the teacher who sets the purpose, and not an external agent, as is the case for interim/bench-mark assessments." [38, p. 4-5]. The use of rubric is a common method to conduct classroom assessment is through the use of rubrics. Rubrics inform students about the learning objectives and provide teachers with clear guidelines to make the grading process less subjective [39]. Toh et al. designed a practical worksheet and an accompanying assessment rubric for the MProSE project which focused on teaching about problem solving [12]. The use of a rubric allows the most valued processes to be assessed in addition to the correctness of the solution of the task. For example, the practical worksheet designed to encourage students to use Polya's four-step model and can be used when teaching

for/through problem solving by Toh et al. assesses both the processes and product of problem solving, in view of the fact that the processes of problem solving are equally valuable as the final product.

Problem Openness and Complexity

The openness of mathematics tasks can be characterised based on five task variables: goal, method, complexity, answer, and extension [40]. Problem solving tasks are characterised by the openness in their solutions and extensions, which allows for the last stage in Polya's four-step model (i.e., check and expand). They are also characterised by a closed goal and answer as students are given a specific problem to investigate which has only one answer. However, problem solving tasks can differ in their openness based on their task complexity.

Teachers need to be aware of the type of complexity possessed by the designed problem. Problem solving tasks should not be closed in complexity. Caution should be taken by teachers when providing scaffolding through the design of the problem or during the problem solving process, especially if the complexity of the designed problem is subject-dependent. The expertise reversal effect is the need to modify instructional methods and levels of instructional guidance to changing levels of learner expertise during a learning session [41].

Learners' Cognition and Affection

Teachers need to know their learners (both cognitive and affective) in their problem design. Mathematical cognition refers to the processes by which individuals come to understand mathematical ideas and individuals' cognitive abilities can be at different levels [42]. Chamberlin defines learner's affect as their beliefs, attitudes, and emotions towards a subject [43]. In the context of mathematical problem solving, being aware of and in control of one's emotions can enhance the likelihood of them finding a solution [44]. In other words, student's attitudes and confidence towards problem solving is crucial to solving the designed problem.

Teachers can use differentiated instruction to respond to the cognitive and affective variance among students [45]. Tomlinson states that "In a differentiated classroom, the teacher proactively plans and carries out varied approaches to content, process, and product in anticipation of and response to student difference in readiness, interest, and learning needs." [46, p. 7]. Teachers can differentiate by providing different levels of scaffolding based on the students' readiness and learning needs, or even modify the problem to cater to students' cognitive and affective needs. Anthony and Hunter proposed the following practices to empower students through cognitive and affective processes:

- i) student should engage in rich mathematical discourse through group activities
- ii) teacher should notice and value of their students' thinking and use of their thinking as a resource for learning
- iii) teachers should position their students as competent [47].

Lesson Enactment

Teachers' lesson enactment is largely influenced by "students' interests and experience, instructional strategies, curriculum resources, teacher's pedagogical beliefs, practice and expertise, parental expectations, school organization, community and culture, high-stakes examinations, curriculum policies, and so forth" [48, p. 270].

A typical lesson often comprises of one or more cycles of instruction, which comprises combinations of Development (D), Student Work (S) and Review (R), based on the lesson objectives [49]. We propose that when teaching *for* problem solving, the lesson enactment could typically follow the sequence of D-S-R. However, when teaching *through* problem solving, the lesson enactment could follow the sequence of D-R-S. For both cases, teachers need to ensure that there is sufficient lesson time for the cycle of instruction to be completed such that the instructional objectives are achieved.

CONCLUSION

Mathematical problem solving resides at the heart of the mathematical curriculum in many countries in the world. This paper hopes to contribute to the ongoing efforts to incorporate mathematical problem solving into classrooms by addressing the difficulties teachers have faced while doing so. In the discussion above, it is clear to the reader that the enactment of problem solving is very much tied to the task design itself. However, readers should be cautioned that the guiding principles we propose have not been tested in mathematics classrooms. Hence, further research needs to be carried out in order to validate and improve on the proposed principles.

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REFERENCES

- National Council of Teachers of Mathematics (NCTM): An Agenda for Action: Recommendations for School Mathematics in the 1980s. Reston, VA: NCTM. (1980).
- [2] Department for Education: National curriculum in England: mathematics programmes of study. GOV.UK. Retrieved from https://www.gov.uk/government/publications/national-curriculum-in-england-mathematicsprogrammes-of-study/national-curriculum-in-england-mathematics-programmes-of-study. (2013, September 11).
- [3] Australian Association of Mathematics Teachers (AAMT). Joint Statement On Proposed Maths Curriculum: Why Maths Must Change. Australian Association of Mathematics Teachers. Retrieved from https://aamt.edu.au/wpcontent/uploads/2020/07/earlymaths.pdf. (2021, March 31).
- [4] Wilkerson, T.: Problem Solving: An Approach to Understanding and Critiquing Our World. National Council Of Teachers Of Mathematics. Retrieved from https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Trena-Wilkerson/Problem-Solving_-An-Approach-to-Understanding-and-Critiquing-Our-World/. (2022, July).
- [5] Ofsted: Research review series: mathematics. GOV.UK. Retrieved from https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-seriesmathematics#curriculum-sequencing-conditional-knowledge.(2021, May 25).
- [6] Ministry of Education (MOE): Education Statistics Digest 2021. Singapore: Ministry of Education, Research and Management Information Division, Management Information Branch. (2021).
- [7] Fan, L. H., & Zhu, Y.: From convergence to divergence: the development of mathematical problem solving in research, curriculum, and classroom practice in Singapore. ZDM, 39(5), 491–501. (2007).
- [8] Ministry of Education (MOE): Mathematics syllabus (Lower Secondary). Singapore: Curriculum Planning Division. (1990).
- [9] Ministry of Education (MOE): Secondary mathematics syllabus. Singapore: Ministry of Education, Curriculum Planning and Development Division. (2019).

- [10] Kaur, B., & Yeap, B. H.: Mathematical Problem Solving in Singapore Schools. In B. Kaur, B. H. Yeap & M. Kapur (Eds.), Mathematical problem solving (pp. 1-13). Singapore: World Scientific. (2009).
- [11] Toh, T. L., Tay, E. G., Leong, Y. H., Quek, K. S., Ho, F. H., Dindyal, J., & Toh, P. C.: Mathematical Problem Solving for Everyone: Infusion and Diffusion. In Y. H. Leong, E. G. Tay, K. S. Quek, T. L. Toh, P. C. Toh, J. Dindyal, F. H. Ho, & R. A. S Yap. (Eds.), Making mathematics more practical: Implementation in the schools (pp. 1-21). Singapore: World Scientific. (2014).
- [12] Toh, T. L., Quek, K. S., Leong, Y. H., Dindyal, J., & Tay, E. G.: Making mathematics practical: An approach to problem solving. Singapore: World Scientific. (2011).
- [13] Leong, Y. H., Lu, P. C., Toh, W. Y, K.: Use of Challenging Items in Instructional Materials by Singapore Secondary School Mathematics Teachers. In B. Kaur & Y. H. Leong (Eds.), Mathematics Instructional Practices in Singapore Secondary Schools (pp. 231 – 248). Springer. (2021).
- [14] Leong, Y. H., Toh, T. L., Tay, E. G., Toh, P. C., Quek, K. S., Dindyal, J., & Ho, F. H.: Another Step towards Mathematical Problem Solving for Everyone. In Y. H. Leong, E. G. Tay, K. S. Quek, T. L. Toh, P. C. Toh, J. Dindyal, F. H. Ho, & R. A. S Yap. (Eds.), Making mathematics more practical: Implementation in the schools (pp. 107-114). Singapore: World Scientific. (2014).
- [15] Leong, Y. H., Tay, E. G., Toh, T. L., Quek, K. S., Toh, P. C., & Dindyal, J.: Infusing Mathematical Problem Solving in the Mathematics Curriculum: Replacement Units. In P. Felmer, E. Pehkonen, & J. Kilpatrick (Eds.), Posing and Solving Mathematical Problems: Advances and New Perspectives (pp. 309-325). Springer. (2016).
- [16] Leong, Y. H.: Problems of teaching in a Singapore reform-oriented mathematics classroom. Lambert Academic Publishing. (2009).
- [17] Mason, J., & Johnston-Wilder, S.: Designing and using mathematical tasks. United Kingdom: Tarquin Publications. (2006).
- [18] Stein, M. K., Grover, B. W., & Henningsen, M.: Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. American Educational Research Journal, 33(2), 455–488. (1996).
- [19] Polya, G.: How to solve it. Princeton: Princeton University Press. (1945).
- [20] Lester, F. K.: Trends and issues in mathematical problem-solving research. In R. Lesh & M. Landau (Eds.), Acquisition of mathematical concepts and processes (pp. 229-261). Orlando, Fl: Academic Press. (1983).
- [21] Schoenfeld, A.: Problem solving in the mathematics curriculum: A report, recommendations, and an annotated bibliography. Mathematical Association of America, Committee on the Teaching of Undergraduate Mathematics. (1983).
- [22] Schoenfeld, A.: Mathematical problem solving. Orlando, Fl: Academic Press. (1985).
- [23] Schroeder, T., & Lester, F.: Developing understanding in mathematics via problem solving. In P. Traffon & A. Shulte (Eds.), New directions for elementary school mathematics: 1989 yearbook (pp. 31-42). Reston, VA: NCTM. (1989).
- [24] Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M.: Elementary and middle school mathematics: teaching developmentally (10th ed.). Pearson. (2019).
- [25] Lester, F. K., & Charles, R. Teaching Mathematics Through Problem Solving: Pre-K-Grade 6. Reston, VA: National Council of Teachers of Mathematics. (2003).
- [26] Stein, M., Kinder, D., Silbert, J., & Carnine, D. W.: Designing Effective Mathematics Instruction: A Direct Instruction Approach (4th ed.). Pearson Merrill Prentice Hall. (2006).
- [27] Yoong, W. K.: Effective mathematics lessons through an eclectic Singapore approach. Singapore: World Scientific. (2015).
- [28] Butt, G.: Lesson planning (3rd ed.). Continuum International Publishing Group. (2008).
- [29] Anderson, L. W., & Krathwohl, D. R. (Eds.).: A taxonomy for learning, teaching and assessment: A revision of Bloom's taxonomy of educational objectives (complete ed.). Longman. (2001).
- [30] Houff, S.: Instructional alignment: Optimizing objectives, methods, and assessment for developing unit plans. Rowman & Littlefield. (2012).
- [31] Piaget, J.: The origins of intelligence in children. (M. Cook, Trans.) New York: Norton. (1952).
- [32] Matlin, M. W.: Cognitive psychology (8th ed.). John Wiley & Sons. (2014).
- [33] Tan, O. S., Chye, Y. L. S., Lim, K. J., Chua, B. L., Tuckman, B. W., & Monetti, D. M.: Educational psychology: An asia edition. Cengage Learning Asia Ltd. (2017).
- [34] Vygotsky, L. S.: Mind in society: The development of higher psychological processes (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Trans.). Cambridge, MA: Harvard University Press. (1978).
- [35] Hino, K.: Developing Interaction Toward the Goal of the Lesson in a Primary Mathematics Classroom. In Toh, P. C., & Chua, B. L. (Eds.), Mathematics instruction: Goals, tasks and activities (pp. 133-158). Singapore: World Scientific. (2018).
- [36] Hill, H., Rowan, B., & Ball, D.: Effects of teachers' mathematical knowledge for teaching on student achievement. American Educational Research Journal, 42, 371-406. (2005).
- [37] De Lange, J.: Large-scale assessment and mathematics education. In F. K. Lester, Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 1111 – 1142). Charlotte, NC: Information Age Publishing. (2007).
- [38] Brookhart, S. M., & McMillan, J. H.: Classroom assessment and educational measurement. Routledge. (2020).
- [39] Depka, E. M.: Designing assessment for mathematics. SAGE Publications. (2007).

- [40] Yeo, J. B. W.: Development of a Framework to Characterise the Openness of Mathematical Tasks. International Journal of Science and Mathematics Education, 15(1), 175–191. (2015).
- [41] Sweller, J., Ayres, P., & Kalyuga, S.: Cognitive Load Theory. Springer. (2011).
- [42] Gilmore, C., Gobel, S. M., & Inglis, M.: An introduction to mathematical cognition. Routledge. (2018).
- [43] Chamberlin, S. A.: The Construct of Affect in Mathematical Modeling. In Chamberlin, S. A. & Sriraman, B. (Eds.), Affect in mathematical modeling. Springer. (2019).
- [44] Chamberlin, S. A., & Parks, K.: A comparison of student affect after engaging in a mathematical modeling activity. International Journal of Education in Mathematics, Science and Technology (IJEMST), 8(3), 177-189. (2020).
- [45] Tomlinson, C. A.: The Differentiated Classroom: Responding to the Needs of All Learners (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development. (2014).
- [46] Tomlinson, C. A.: How to differentiate instruction in mixed-ability classrooms (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development. (2001).
- [47] Anthony, G., & Hunter, R.: Developing Student Voice in the Mathematics Classroom. In B. Kaur & N. H. Lee (Eds.), Empowering Mathematics Learners (pp. 99-115). Singapore: World Scientific. (2017).
- [48] Deng, Z., Gopinathan, S., & Lee, C. K. E.: The Singapore Curriculum: Convergence, Divergence, Issues and Challenges. In Z. Deng, S. Gopinathan, & C. K. E. Lee (Eds.), Globalization and the Singapore Curriculum : From Policy to Classroom (pp. 263-275). Springer. (2013).
- [49] Kaur, B., Tay, E. G., Tong, C. L., Toh, T. L., & Quek, K. S.: The Instructional Core That Drives the Enactment of the School Mathematics Curriculum in Singapore Secondary Schools. In B. Kaur & Y. H. Leong (Eds.), Mathematics Instructional Practices in Singapore Secondary Schools (pp. 45-59). Singapore: World Scientific. (2021).

http://jurnalnasional.ump.ac.id/index.php/Dinamika