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A Cognitive Cascade Analysis of High School Students' Problem-Solving Difficulties in Algebraic Derivatives

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ABSTRACT

Mathematics instruction at the high school level generally continues to emphasize procedural mastery, so students' problem-solving abilities are often assessed based on the accuracy of their final answers. Previous studies have tended to focus on procedural errors or students' success rates, while studies that reveal the complete and meaningful process of students' mathematical problem solving remain limited. Therefore, this study aims to explore students' mathematical problem-solving processes in the topic of algebraic derivatives. This study employed a qualitative case study design with 32 twelfth-grade students. Data were collected through a written mathematical problem-solving test administered to all participants, followed by semi-structured interviews with a purposively selected group of students representing different problem-solving profiles. The data were analyzed thematically using NVivo to identify patterns in problem interpretation, mathematical modeling, solution strategies, mathematical communication, and the meaningful use of mathematics. The results showed that students' main difficulty occurred at the mathematical modeling stage, where many students were unable to translate contextual information into appropriate algebraic representations before applying differentiation procedures. This difficulty triggered a chain reaction in subsequent stages, leading to the use of mechanical problem-solving strategies and affecting students' mathematical communication and meaningful use of mathematics. This study confirms that mastery of derivative procedures does not guarantee meaningful problem solving. The findings imply the need for learning approaches that emphasize conceptual understanding, mathematical modeling, and reflection. They also open opportunities for further research based on learning interventions.

Keywords: Derivatives, High School Mathematics, Mathematical Modeling, Problem Solving, Qualitative Study.

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Introduction

Mathematics education at the high school level plays a fundamental role in developing higher-order mathematical thinking skills, particularly reasoning-based problem-solving (Singh et al., 2024), representation (Putra et al., 2023), and concept generalization (Zhou et al., 2024). Among mathematical topics taught at this level, differential calculus, particularly derivatives, requires students to coordinate these competencies extensively (Rathour et al., 2023). Understanding derivatives involves more than executing symbolic procedures; students are expected to understand relationships among variables, interpret changes mathematically, and construct meaningful representations of problem situations (Galindo Illanes et al., 2025). Consequently, evaluating students' success in calculus should extend beyond

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computational accuracy and include their ability to construct and apply mathematical meaning throughout the problem-solving process.

Mathematical problem-solving is widely regarded as the core of mathematical activity and one of the principal objectives of mathematics education (Felmer, 2023; Sinaga et al., 2023). In addition to supporting mathematical achievement, problem-solving contributes to the development of higher-order thinking skills. It strengthens students' capacity for logical reasoning, decision-making, and systematic analysis in academic and real-world contexts (Rohyati & Purwanto, 2023). According to Saputra et al. (2024), mathematical problem-solving is the process of generating solutions by applying previously acquired mathematical knowledge. To systematically examine students' mathematical thinking, this study adopts George Pólya's problem-solving framework, consisting of understanding the problem, devising a plan, carrying out the plan, and looking back (Polya, 1945). This framework provides a structured perspective for examining how students interpret problems, formulate representations, implement strategies, and evaluate mathematical outcomes.

Despite the importance of these competencies, previous studies indicate that students continue to experience considerable difficulties in derivative problem-solving (Bibi et al., 2025; Díaz, 2024). These difficulties extend beyond computational procedures and include limitations in mathematical modeling, selecting appropriate strategies, and communicating mathematical reasoning coherently (Muhaimin et al., 2024; Sa'diyah et al., 2024). In many cases, students can differentiate functions symbolically but struggle to interpret contextual situations, translate them into mathematical representations, or explain the meaning of the solutions obtained. Such conditions indicate a discrepancy between the intended goals of calculus learning and instructional practices that continue to emphasize procedural execution rather than conceptual understanding and meaningful reasoning.

Among these difficulties, mathematical modeling appears particularly critical because it functions as a cognitive bridge connecting contextual understanding and symbolic mathematical reasoning. Difficulties occurring at this stage may interrupt students' transition from interpreting situations to constructing mathematical representations. When students fail to formulate appropriate mathematical models, subsequent stages of problem-solving often become fragmented, influencing strategy selection, mathematical communication, and interpretation of results. This propagation pattern suggests a hierarchical cognitive cascade in which difficulties emerging at earlier stages influence subsequent cognitive processes. To investigate these processes more comprehensively, students' written responses become important because they reveal not only final solutions but also students' reasoning processes, strategy selection, and

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conceptual understanding (Maclean & Bayley, 2024; Park et al., 2023). Therefore, thematic analysis assisted by NVivo was employed to systematically organize and interpret these qualitative patterns.

Understanding students' difficulties in derivative problem-solving requires not only procedural analysis but also examination of the cognitive structures underlying mathematical thinking. Fragmented problem-solving processes may emerge from incomplete conceptual constructions that prevent students from integrating mathematical ideas meaningfully. In this context, APOS theory conceptualizes mathematical understanding through four interconnected cognitive structures: Action, Process, Object, and Schema (Arnon et al., 2014; Dubinsky, 1991; Syaiful et al., 2025). Students operating predominantly at the action level tend to rely on procedural manipulation, whereas meaningful mathematical understanding requires schema-level integration involving interpretation, mathematical modeling, procedural reasoning, and conceptual understanding. Thus, APOS theory provides an appropriate analytical lens for interpreting the cognitive mechanisms underlying students' difficulties in derivative learning.

Although research on calculus learning difficulties has been widely conducted, previous studies have primarily focused on procedural errors, misconceptions, or quantitative learning outcomes (Kramer et al., 2023; Lin et al., 2025). Studies specifically examining the interrelationships among stages of mathematical problem-solving, particularly how modeling difficulties propagate across subsequent cognitive stages, remain limited. Furthermore, research integrating problem-solving frameworks with qualitative thematic analysis to explain fragmented mathematical reasoning has not been extensively developed (Dönmez & Akkoç, 2025). Unlike previous studies that investigated students' difficulties in isolation, this study examines derivative problem-solving as an interconnected cognitive process by integrating Pólya's problem-solving framework, APOS theory, and qualitative thematic analysis. Accordingly, this study seeks to: (1) identify and categorize students' difficulties at each stage of George Pólya's problem-solving framework in algebraic derivative problems; (2) analyze how difficulties in mathematical modeling propagate across subsequent stages of problem-solving; and (3) construct an interpretive explanation of the cognitive mechanisms underlying fragmented mathematical problem-solving processes. The findings are expected to contribute both theoretically to the understanding of the structure and dynamics of mathematical problem-solving and practically to the development of instructional strategies that strengthen conceptual understanding and mathematical modeling.

Methods

Research Design

This study employed an exploratory qualitative case study design to investigate students' mathematical problem-solving processes in solving algebraic derivative problems. A qualitative approach was selected because the study aimed to understand students' reasoning processes, problem-solving experiences, and patterns of mathematical thinking rather than to measure learning outcomes quantitatively. The exploratory case study design enabled an in-depth examination of how students interpreted problems, constructed mathematical representations, selected solution strategies, communicated mathematical ideas, and used mathematics meaningfully within a specific educational context (Türkoğlu & Yalçınalp, 2024). This design was considered appropriate because the study aimed not only to describe students' observable responses but also to explain the cognitive mechanisms underlying fragmented mathematical reasoning. Therefore, the study focused on understanding how students' difficulties emerged and propagated across interconnected stages of mathematical problem-solving.

Participants

The participants consisted of 32 twelfth-grade students (N=32) from SMAN 1 Jatitujuh, a public high school in West Java, Indonesia. Participants were selected using purposive sampling based on two criteria. First, they had received formal instruction on derivatives as part of the national mathematics curriculum. Second, they had reached an educational level at which symbolic reasoning, algebraic manipulation, and introductory calculus concepts were expected to have developed. All participants completed a written mathematical problem-solving test, which served as the primary source of data for identifying broad patterns in students' mathematical reasoning. From this group, 10 students were purposively selected for follow-up semi-structured interviews. Selection of interview participants was based on variation in problem-solving profiles identified from written responses, including students demonstrating coherent problem-solving processes, students showing procedural success accompanied by conceptual inconsistency, and students exhibiting persistent difficulties across multiple stages of problem-solving. This strategy was intended to ensure analytical variation and capture a broader range of cognitive characteristics in students' derivative problem-solving. The study was conducted in a regular classroom setting, where derivatives were taught according to the standard national curriculum. This context was considered suitable for examining students' engagement with derivative problems under ordinary instructional conditions, particularly regarding problem interpretation, mathematical modeling, and symbolic reasoning.

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Instruments
The primary instruments in this study were a written mathematical problem-solving test and semi-structured interviews. In qualitative research, the researcher also served as the principal research instrument responsible for data collection, coding, interpretation, and thematic synthesis. The written test consisted of open-ended derivative problems designed to explore students' mathematical thinking processes. The tasks were developed to assess students' abilities across problem-solving stages, including problem interpretation, mathematical modeling, solution strategy selection, mathematical communication, and meaningful interpretation of results. Students were also required to provide written explanations of their solution procedures to capture traces of their mathematical reasoning.

To establish content validity, the written instrument was evaluated by two experts: a mathematics lecturer from Majalengka University and an experienced high school mathematics teacher. The evaluation focused on three aspects: alignment with derivative learning objectives, appropriateness of cognitive demands for twelfth-grade students, and clarity of language and contextual representation. Several revisions were subsequently implemented based on expert feedback to improve linguistic clarity and strengthen consistency across contexts and derivative concepts. Before implementation, the instrument was pilot-tested with students outside the main participant group to evaluate readability, clarity, and difficulty level. Minor revisions were subsequently made based on the pilot findings to improve comprehension while maintaining the tasks' intended cognitive demands.

Semi-structured interviews were conducted to obtain deeper explanations of students' written responses. The interview protocol focused on students' interpretations of derivative problems, reasoning processes, strategy selection, and understanding of the meaning of mathematical results. This approach ensured consistency across participants while allowing flexibility to explore individual reasoning in greater depth.

Data Collection
Data collection was conducted in two stages using written tests and semi-structured interviews. During the first stage, all 32 students independently completed the written mathematical problem-solving test during regular classroom activities. Students were instructed to document all solution procedures and provide explanations that reflect their reasoning. This stage generated the primary dataset for identifying patterns in students' mathematical problem-solving. During the second stage, semi-structured interviews were conducted with 10 selected participants. The interviews aimed to clarify students' written responses, investigate the reasons underlying their procedural choices, and identify difficulties encountered during problem-solving.

Each interview lasted approximately 10–20 minutes and was conducted individually in a quiet area within the school environment. With participants' consent, all interviews were audio-recorded and transcribed verbatim for analysis. Interview transcripts were subsequently compared with corresponding written responses to support cross-source interpretation and thematic validation.

Data Analysis

Data were analyzed using thematic analysis, assisted by NVivo software, to identify patterns, themes, and meanings that represent students' mathematical problem-solving processes. The analysis included students' written responses and interview transcripts. The analytical process began with data familiarization through repeated reading of written responses and interview transcripts to develop a comprehensive understanding of the data. Initial coding followed a hybrid approach that combined inductive coding with theoretically informed coding guided by Pólya's problem-solving framework and emerging patterns identified in the data. Codes were assigned to meaningful segments representing students' cognitive activities, including problem interpretation, identification of relevant information, mathematical modeling, strategy selection, procedural implementation, mathematical communication, conceptual interpretation, and meaningful mathematical use.

The initial codes were subsequently organized into broader categories through repeated comparison of similarities and differences among coded segments. NVivo node hierarchy features were used to establish relationships among codes and categories. Through an iterative analytical process, categories were refined into broader themes representing dominant patterns and underlying cognitive characteristics in students' mathematical problem-solving processes. To strengthen interpretive depth, findings from written responses and interview data were analyzed comparatively so that interview results could confirm, clarify, and extend patterns identified in students' written responses. Findings were then presented through thematic descriptions supported by representative excerpts from students' written work and interview transcripts to maintain clear connections between empirical evidence and interpretation.

Trustworthiness

Several procedures were employed to strengthen the credibility and trustworthiness of this study. First, methodological triangulation was conducted by comparing students' written responses with interview data to strengthen interpretive accuracy across both conceptual and procedural dimensions. Second, an audit trail was maintained throughout coding and analysis procedures by systematically documenting coding decisions, node development, category refinement, and thematic interpretation within NVivo. Third, peer discussions with colleagues experienced in qualitative research and mathematics education were conducted to evaluate coding consistency, thematic coherence, and interpretive validity. Finally, representative excerpts from written responses and interview transcripts were included in the findings to ensure interpretive transparency and maintain strong connections between analytical claims and empirical evidence. Collectively, these procedures were intended to strengthen the credibility, dependability, and transparency of the study.

Result and Discussions

Result

Initial Overview of NVivo-Based Analysis

Before discussing the main themes in depth, this section provides an initial overview of the data analysis results generated through NVivo-assisted coding. This preliminary stage aimed to establish an analytical context for the scope of the data, the coding structure, and the patterns of coded references that formed the basis for developing the research themes. The analyzed data consisted of students' written responses to mathematical problem-solving tests and transcripts of semi-structured interviews. All data were coded using a hybrid coding approach that combined theoretically informed coding based on Pólya's problem-solving framework with patterns emerging from the data. The resulting initial codes were subsequently organized into broader categories and themes.

The preliminary analysis indicated that the coding process yielded 10 primary nodes, which were subsequently grouped into 5 conceptual themes. Each node represented a specific aspect of students' mathematical problem-solving processes, ranging from initial problem interpretation to the meaningful use of mathematics. Variations in the distribution of coded references across nodes reflected differences in students' problem-solving patterns and cognitive characteristics.

Table 1 shows that nodes related to initial problem interpretation and the application of procedural strategies were the most frequently observed in the data. In contrast, nodes associated with mathematical communication and the meaningful use of mathematics appeared less frequently. This pattern suggests that although many students were able to initiate the problem-solving process, they experienced increasing difficulties in the later stages, which required conceptual integration and reflective reasoning. The distribution of nodes further indicates that mathematical modeling functioned as a critical transition stage within students' problem-solving processes. Nodes related to students' inability to formulate algebraic models were frequently associated with difficulties in strategy implementation and limitations in explaining mathematical results. These relationships suggest that difficulties emerging at earlier stages tended to influence subsequent stages of problem-solving. Based on these interrelationships, an integrated hierarchical cognitive cascade model was developed to visualize the progression of students' difficulties across interconnected stages of mathematical problem solving, as shown in Figure 1.

Table 1. The nodes and conceptual themes identified through the NVivo

Conceptual Theme	Node	Data Source	Analysis Focus	N
Problem Interpretation	Able to identify	Test & interview	The ability to recognize information and the demands of a problem	17
	Unable to identify	Test and interview	Difficulty understanding the given and required elements	15
Mathematical Modeling	Able to formulate and develop models	Test and interview	Transforming problems into algebraic models	4
	Unable to formulate and develop a model	Test and interview	Obstacles in symbolic representation	28
Solution Strategy	Able to apply strategies	Test and interview	Selection and implementation of derivative procedures	15
Solution Strategy	Unable to implement the strategy	Test and interview	Strategic and procedural errors	17
Mathematical Communication	Able to explain responses clearly	Test and interview	Ability to explain processes and results	4
Mathematical Communication	Unable to explain responses clearly	Test and interview	Limitations of communication and reflection	28
Meaningful Use of Mathematics	Able to use mathematics meaningfully	Test and interview	Integration of concepts and problem contexts	2
Meaningful Use of Mathematics	Unable to use mathematics meaningfully	Test and interview	Problem solving was mechanistic	30

To synthesize the findings derived from the five thematic analyses, an integrated hierarchical cognitive cascade model was developed. Figure 1 illustrates how students' cognitive difficulties progressed across interconnected stages of mathematical problem-solving, from problem interpretation to meaningful mathematical use. The dominant pathway represents the progression of cognitive difficulties experienced by most students, particularly those originating from weaknesses in problem interpretation and mathematical modeling. Difficulties emerging at these earlier stages frequently carried over into subsequent stages, influencing strategy selection, mathematical communication, and the interpretation of results. In contrast, an alternative pathway represents students who were able to integrate problem interpretation, mathematical modeling, strategy use, communication, and conceptual understanding into coherent and meaningful mathematical reasoning.

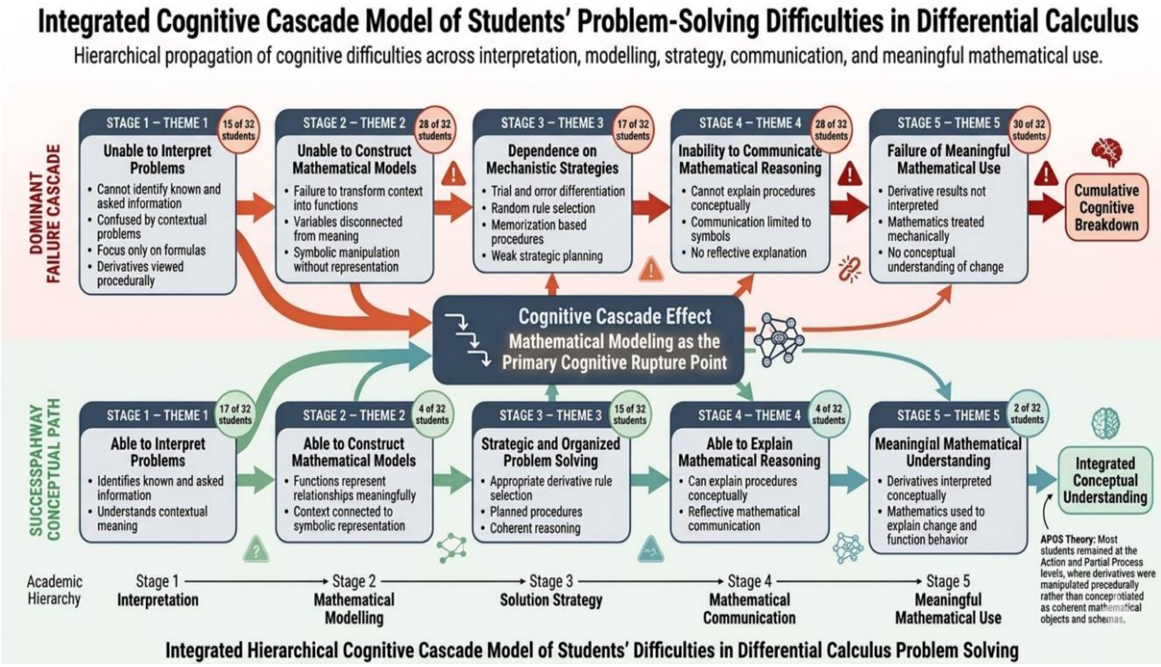
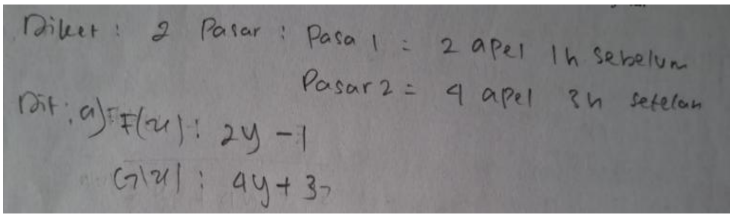


Figure 1. The Cognitive Cascade Model of Students' Difficulties in Differential Calculus

Theme 1. Students' Initial Interpretation of Function Derivative Problems

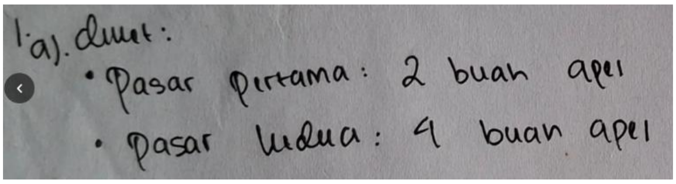
Students' ability to interpret problems represents an important initial stage that influences the overall quality of mathematical problem-solving processes (Satiti & Wulandari, 2021). The analysis showed that students' initial interpretations of algebraic derivative problems varied considerably and revealed two dominant patterns: students who were able to identify essential problem elements and students who experienced difficulties during the initial identification stage. These differences extended beyond procedural variation and influenced students' subsequent problem-solving processes. Within the broader cognitive cascade framework presented in Figure 1, students' initial interpretation patterns are further illustrated by the contrasting responses shown in Figures 2 and Figure 3. Students who successfully identified relevant problem information generally demonstrated more coherent modelling and solution processes, whereas students who experienced difficulties during early interpretation frequently encountered difficulties across later stages of problem-solving, including modelling, strategy implementation, communication, and meaningful mathematical use.



It is known: 2 markets
Market 1 = 2 apples 1 day before
Market 2 = 4 apples 3 days later

Figure 2. Example of answers from students who are able to identify problems

The analysis of students' written responses revealed clear differences in how students initiated the problem-solving process. As shown in Figure 2, students who successfully identified problem elements generally wrote down relevant information and explicitly stated what was being asked before proceeding to subsequent solution stages. Although this identification remained relatively literal, it provided an initial basis for further problem-solving activities.



It is known:
Market 1 = 2 apples
Market 2 = 4 apples

Figure 3. Example of answers from students who are unable to identify problems

Conversely, as shown in Figure 3, students who had difficulty identifying problem elements tended to write symbols, formulas, or derivative operations immediately, without demonstrating an understanding of the problem context. Responses in this group frequently skipped the interpretation stage and approached the problem primarily as a procedural task. This pattern suggests that difficulties that emerged during the initial stage were associated not with limited knowledge of derivative procedures but with challenges in understanding the problem's meaning and purpose.

Interview findings further indicated that differences in students' initial interpretive abilities were related not only to their understanding of the language used in the problem but also to their interpretation of derivative concepts. Students who successfully identified problem elements generally recognized that derivatives were associated with changes in functions, although this understanding remained partial and largely procedural.

Researcher	:	*"How do you usually begin solving a derivative problem?"*
Student RNA	:	*"I usually look at the given function and what is being asked. If I am asked to find the derivative, I know I have to differentiate the function, but sometimes I do not really understand why the function has to be*

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differentiated at a certain point. I focus more on making sure the steps are correct."

This quote shows that the student had linked the problem to the concept of function derivatives. However, their understanding remained oriented toward what to do rather than why the steps were mathematically relevant. Derivatives were perceived as a symbolic procedure rather than as a representation of change or the slope of a function.

In contrast, students in the group who were unable to identify problems showed a very limited conceptual understanding of function derivatives. Difficulties arose not only at the stage of reading the question but also in their inability to relate its context to the meaning of derivatives.

Researcher	:	*"What comes to your mind when you encounter a derivative problem?"*
Student PNA	:	*"When it comes to derivatives, I just think about the formula. I do not really understand what the function means, as long as it is differentiable. If the question uses a story or a rather long function, I get confused about where to start."*

Furthermore, several students stated they were unable to distinguish the purposes of using derivatives, such as determining the gradient, the rate of change, or the properties of a function.

Researcher	:	*"How do you decide what to do when solving a derivative problem?"*
Student RKA	:	*"I know that derivatives are used to find gradients, but if the question does not mention the gradient, I am unsure what to do. Usually, I just differentiate it anyway."*

These findings show that limitations in initial interpretation were closely related to fragmented conceptual understanding, where students were not yet able to connect the concept of derivatives to various representations and mathematical objectives.

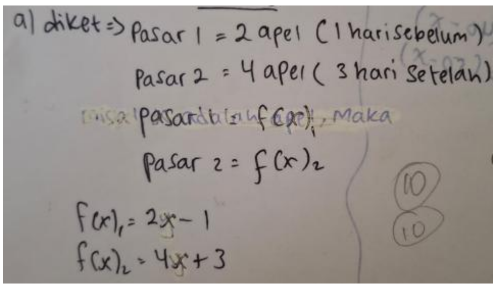
Analytically, students' initial interpretive abilities remained dominated by declarative understanding rather than relational understanding. Students were able to identify the known and required information, but had not yet linked it to the concept of function derivatives in a meaningful way. As a result, the interpretation stage did not fully function as a bridge to appropriate mathematical modelling.

For students who failed to identify the problem, the absence of initial interpretation caused them to move directly to the procedural stage without a strong conceptual basis. This explains the strong relationship between the *node's inability to identify* and difficulties in the modelling and solution strategy stages. Thus, problem interpretation acted as an initial cognitive filter that determined the direction and quality of students' problem-solving processes.

Based on the results of the NVivo analysis, students' written answers, and semi-structured interviews, it can be concluded that the initial interpretation of function-derivative problems was a critical point in students' mathematical problem-solving. The ability to identify the elements of a problem did not guarantee successful problem solving, but failure at this stage almost always led to further procedural and conceptual errors. These findings confirm that the quality of problem-solving was not determined solely by mastery of formulas, but also by students' ability to construct mathematical meaning from the early stages of problem-solving. Consequently, difficulties in initial interpretation constrained students' ability to construct meaningful mathematical models in subsequent stages of problem solving.

Theme 2. Students' Difficulties in Mathematical Modeling of Derivative Function Material

Following the initial interpretation stage, mathematical modelling emerged as an important phase influencing students' progression in mathematical problem-solving (Tasarib et al., 2025). The analysis showed that although some students were able to identify known and required information, mathematical modelling represented the most substantial difficulty encountered in solving algebraic derivative problems. At this stage, students were required to transform contextual information into mathematical representations that accurately reflected relationships among variables.

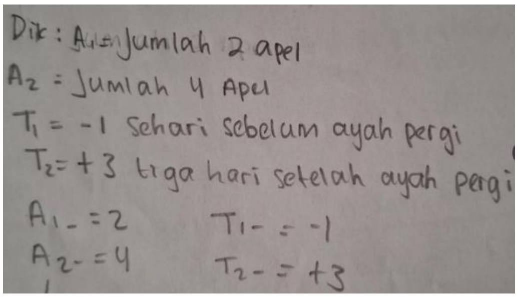


It is known:
Market 1 = 2 apples (1 day before)
Market 2 = 4 apples (3 days later)
Market 1 = $f(x)_1$
Market 2 = $f(x)_2$
$f(x)_1 = 2x - 1$
$f(x)_2 = 4x + 3$

Figure 4. The answer from a Student Who Successfully Constructed a Mathematical Model

Within the broader cognitive cascade framework presented in Figure 1, students' mathematical modelling patterns are more specifically represented by the contrasting responses shown in Figures 4 and Figure 5. The findings indicated that most students experienced difficulties transforming contextual information into appropriate mathematical models, whereas only a small proportion successfully constructed coherent symbolic representations. Students experiencing modelling difficulties frequently encountered challenges in subsequent stages of problem-solving, including strategy implementation and explaining mathematical results.

The analysis of students' written responses revealed substantial differences in how students constructed mathematical models. As illustrated in Figure 4, students who successfully formulated mathematical models generally represented relevant functions based on the problem context and demonstrated a clear relationship between the constructed model and the purpose of the derivative calculation. Responses in this group showed that mathematical representations were consistently linked to subsequent solution procedures.



It is known:
A_1 = amount 2 apples
A_2 = amount 4 apples
T_1 = -1 the day before dad left
T_2 = +3 three days after father left
$A_1 = 2 \quad T_1 = -1$
$A_2 = 4 \quad T_2 = +3$

Figure 5. An Answer from a Student Who Failed to Construct a Mathematical Model

Conversely, as illustrated in Figure 5, students who experienced difficulties in constructing mathematical models tended to write algebraic expressions without clear justification or apply derivative operations without ensuring that the mathematical representation corresponded appropriately to the problem context. In several cases, students directly copied the function's form from the problem without interpreting it as a representation of the relationships among variables in the situation. Responses in this group suggested that mathematical expressions were frequently treated as isolated symbolic forms rather than as representations of contextual relationships.

The interview results reinforced the findings from the analysis of students' written answers by revealing how students interpreted the modelling process. Students who were able to construct mathematical models explained that they tried to understand the relationships between variables before determining the form of the function to be differentiated.

Researcher : *"How do you construct a mathematical model before differentiating the function?"*

Student MAB : *"I try to understand what the function describes first. Once I have the form of the function, then I derive it, so the steps are clearer."*

This statement shows that the student viewed functions as mathematical representations of problem situations, not merely as algebraic forms to be manipulated. Conversely, students who experienced difficulties with modelling reported being confused about determining the relevant form of the function, even though they generally understood the differentiation procedure.

Researcher : *"What difficulties do you experience when writing the function model?"*
Student NA : *"I know how to derive a function, but I am often confused about how the function should be written. If I make a mistake at the beginning, I usually make mistakes until the end."*

Some students also expressed that the narrative context of the question was a major obstacle in constructing a mathematical model.

Researcher : *"What difficulties do you face when the problem is presented in story form?"*
Student PNA : *"If the question uses a story, I often do not know which part to make into a function."*

These interviews show that modelling difficulties stemmed from students' inability to relate the problem's context to the symbolic representation of algebraic functions. Conceptually, difficulties in mathematical modelling reflected students' limitations in transitioning from verbal or contextual representations to symbolic representations. Students tended to understand functions as algebraic forms to be processed, rather than as mathematical models that represent relationships between variables in a situation.

As a result, the differentiation process was carried out mechanistically without first constructing a meaningful model. This condition explains why errors in the modelling stage were almost always followed by errors in the selection and application of solution strategies. Thus, mathematical modelling served as an essential link between problem interpretation and the application of calculus procedures.

Based on the results of the NVivo analysis, students' written answers, and semi-structured interviews, it can be concluded that mathematical modelling was the most critical stage in solving derivative problems involving algebraic functions. Students' inability to construct appropriate mathematical models caused the problem-solving process to stall or deviate from the outset, despite their having adequate procedural knowledge.

From an APOS perspective, students' difficulties in mathematical modelling indicated that most students were still operating at the action level, applying derivative procedures mechanically without meaningful conceptual development. Although students were able to perform differentiation procedures, they experienced difficulties transforming contextual situations into symbolic function representations. This finding suggests that students had not yet internalized the modelling process into a coherent mental structure. Consequently, functions were treated merely as algebraic expressions rather than as mathematical objects representing relationships between variables.

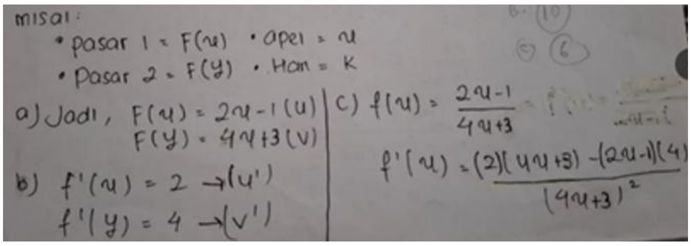
Furthermore, many students appeared unable to transition from process to object construction. They could execute derivative procedures but failed to encapsulate functions as meaningful mathematical models connected to the problem context. As a result, errors in the modelling stage frequently led to inappropriate solution strategies and unsuccessful problem solving. In contrast, students who successfully constructed mathematical models demonstrated process-object coordination by interpreting relationships between variables before applying differentiation procedures. These findings confirm that mathematical modelling requires not only procedural competence but also cognitive construction involving interiorization, encapsulation, and coordination within the APOS framework.

These findings confirm that success in solving mathematical problems was determined not only by mastery of derivative formulas but also by students' ability to represent problem situations in meaningful mathematical models. Difficulties at the modelling stage subsequently influenced students' selection and application of solution strategies, demonstrating the central role of mathematical modelling within the integrated cognitive cascade model. These strategic difficulties are further discussed in Theme 3.

Theme 3. Students' Solution Strategies in Solving Derivative Problems
After constructing mathematical models, students proceeded to the stage of selecting and applying solution strategies. The analysis showed that students' approaches to solving algebraic derivative problems were predominantly procedural, with variations depending on their success during the mathematical modelling stage. Students who successfully constructed appropriate mathematical models generally applied more organized and targeted solution strategies, whereas students who

experienced difficulties in modelling tended to demonstrate inconsistent approaches characterized by trial-and-error procedures.

Within the broader cognitive cascade framework presented in Figure 1, students' strategy patterns are more specifically represented by the contrasting responses shown in Figures 6 and Figure 7. The findings indicate that many students had difficulty determining appropriate solution procedures, even after identifying function forms or performing initial differentiation. In many cases, students relied on memorized formulas or previously encountered examples rather than analyzing the problem structure conceptually. Responses in this group showed that strategy selection frequently remained procedural and lacked consistency across solution stages.



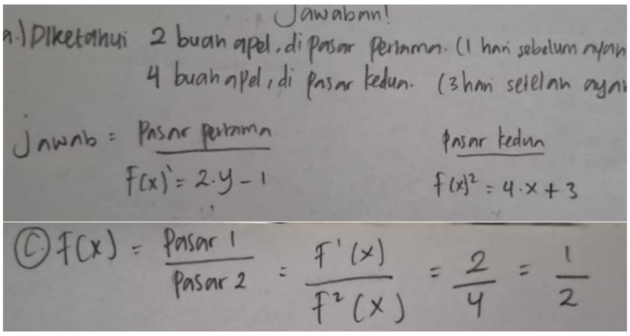
For example:
Market 1 = f(x)   apple = x
Market 2 = f(y)   day=constant
So, f(x) = 2x – 1 (u)
f(x) = 4x +3 1 (v)
f′(x) = 2 → (u′)
f′(x) = 4 → (v′)

$$f(x) = \frac{2x-1}{4x+3}$$

$$f'(x) = \frac{2(4x+3)-(2x-1)(4)}{(4x+3)^2}$$

Figure 6. Able to apply solution strategies

Analysis of students' written responses, presented in Figure 6, shows that students who successfully applied solution strategies generally organized their procedures systematically, beginning with an appropriate mathematical model and applying relevant derivative rules to obtain consistent results. Responses in this group followed a structured sequence of steps and demonstrated a clear relationship between mathematical representations and subsequent solution procedures.



It is known:
2 apples , Market 1 = (1 day before)
4 apples, Market 2 = (3 days later)
Market 1, $f(x)_1 = 2y - 1$
Market 2, $f(x)_2 = 4x + 3$

$$F'(x) = \frac{market\ 1}{market\ 2}$$

$$\frac{f'(x)}{f'(x)} = \frac{2}{4} = \frac{1}{2}$$

Figure 7. Unable to apply solution strategies

Conversely, as illustrated in Figure 7, students who had difficulty applying solution strategies demonstrated inconsistent, less structured solution procedures. In several cases, students applied derivative rules directly without considering whether the

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mathematical model appropriately represented the problem context or whether the procedures aligned with the intended purpose of the calculation. Some responses used multiple derivative rules without clear justification, indicating uncertainty in selecting appropriate procedures. Responses in this group frequently lacked consistency in the sequence and organization of solution steps.

The interview results reinforce the finding that students' solution strategies are strongly influenced by their understanding of function derivatives. Students who applied the strategies explained that they chose solution steps based on their understanding of the purpose of derivative calculations.

Researcher	:	*"What difficulties do you face when the problem is presented in story form?"*
Student ESF	:	*"Once I know the function and model, I usually choose the appropriate derivative rule. So I don't just differentiate randomly, but adjust it to the form of the function."*

This statement shows that the solution strategy is based on the suitability of the function's form to the differentiation rule used. Conversely, students who experienced difficulties with the strategy often reported guessing the steps to take when unsure of the correct procedure.

Researcher	:	*"What do you do when you are unsure which derivative rule to use?"*
Student NR	:	"*I am often confused about which rule to use. So I just try them one by one, sometimes correctly, sometimes incorrectly*."

Some students also revealed that they memorised the steps for solving the problem without understanding the reasons for choosing that strategy.

Researcher	:	*"How do you determine the method you use to solve derivative problems?"*
Student AIY	:	"*I usually remember examples that we have done in class. If it is similar, I use that method, even though it may not necessarily be suitable*."

This interview shows that students' problem-solving strategies tend to rely on procedural memory rather than on conceptual analysis of the problem structure. Conceptually, students' problem-solving strategies reflect a procedural rather than a strategic orientation in thinking. Students understand derivatives as a set of rules that must be applied, but they do not yet view strategy as the result of deliberate problem-solving planning. When mathematical models are poorly constructed, students lack a basis for determining appropriate strategies, leading in inconsistent solution steps. This condition shows that solution strategies cannot be separated from the previous stages. Failure at the mathematical modelling stage directly limits students' ability to design effective strategies. Thus, the solution strategy reflects the quality of students' conceptual and representational understanding of function derivatives.

Based on the results of the NVivo analysis, students' written answers, and semi-structured interviews, it can be concluded that students' solution strategies for solving function-derivative problems remain dominated by procedural approaches that are not always well planned. Students who can construct mathematical models well tend to apply more appropriate and consistent strategies. In contrast, students who experience difficulties with modelling tend to use poorly directed trial-and-error strategies.

These findings confirm that the effectiveness of solution strategies depends heavily on the quality of prior problem interpretation and mathematical modelling. When students relied primarily on procedural approaches, they often struggled in explain and justify their mathematical reasoning coherently. This pattern demonstrates that strategic limitations not only affected procedural accuracy but also constrained students' mathematical communication, which is further discussed in Theme 4.

Theme 4. Students' Mathematical Communication in Conveying the Process and Results of Solutions

Mathematical communication is the final stage that reflects the overall quality of students' problem-solving processes (Teledahl et al., 2025). At this stage, students are expected to explain the solution process and the results obtained coherently, logically, and meaningfully. The analysis results show that students' mathematical communication skills remain very limited, particularly in their ability to relate procedural steps to conceptual reasoning in the context of algebraic function derivatives. This limitation is evident among students who experience difficulties with the interpretation, modelling, and solution-strategy stages.

Within the broader cognitive cascade framework presented in Figure 1, students' mathematical communication patterns are further illustrated by the contrasting responses shown in Figure 8 and Figure 9. The findings indicate that many students had difficulty explaining or justifying the solution procedures they had written. In most cases, students' mathematical communication was limited to presenting final answers or symbolic manipulations without explaining the relationships among the solution steps. Responses in this group frequently showed limited elaboration of

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mathematical reasoning and reduced consistency in explaining the progression of solution procedures.

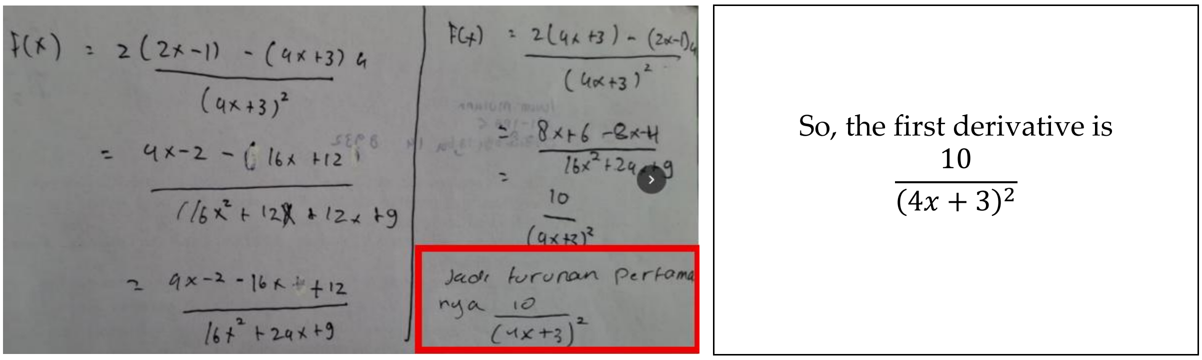


Figure 8. Able to communicate the solution process

Analysis of students' written answers in Figure 8 shows that those who were able to communicate the solution process generally wrote the steps in sequence, accompanied by brief descriptions of the derivative rules used. Although the explanations remained simple, students in this group were able to demonstrate the relationship among mathematical models, strategies, and final results.

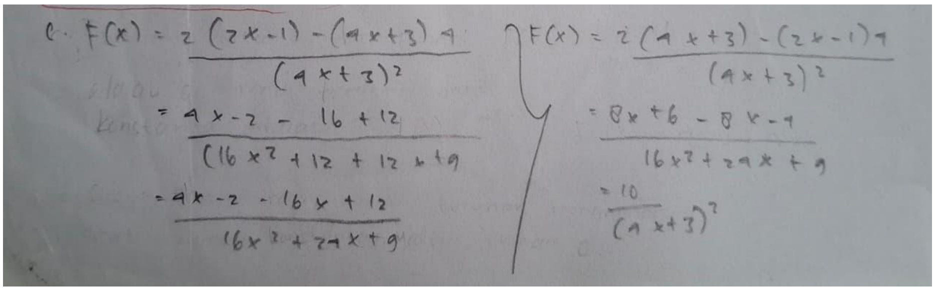


Figure 9. Unable to communicate the solution

Conversely, in Figure 9, students who were unable to communicate their solutions tended to write down a series of symbolic operations without additional explanation. In many cases, students' answers ended with numerical results or derivative forms without any explanation of the purpose or meaning of the steps taken. This pattern shows that students did not yet view mathematical communication as an integral part of problem solving, but rather as an additional, non-essential activity.

The interview results revealed that limitations in mathematical communication are closely related to students' understanding of the concept of function derivatives. Students who explained the solution stated that they tried to understand the reasons for using specific derivative rules.

Researcher : *"When are you able to explain your solution clearly?"*

Student IM : *"If I can explain it, it is usually because I know why I used that rule. So I can recount the steps."*

This statement shows that mathematical communication skills emerge when students have a relatively complete conceptual understanding.

Conversely, students who were unable to explain revealed that they only followed memorised steps without understanding the reasons behind them.

Researcher : *"Why do you find it difficult to explain your solution process?"*
Student RKA : *"I can do it, but if I am asked to explain, I am confused about what to say. I just follow the examples that have been taught."*

Some students also stated that they were not accustomed to being asked to explain their thought processes in mathematics learning.

Researcher : *"Are you usually asked to explain the reasons behind your solution steps?"*
Student TNL : *"We are never asked to explain why we did it that way."*

These interviews show that weak mathematical communication is not only due to individual students' limitations but also to learning practices that emphasise the final result over the thought process.

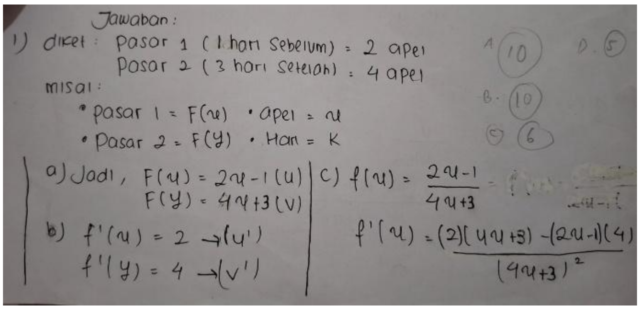
Conceptually, students' mathematical communication is still at the level of symbolic expression and has not yet reached conceptual expression. Students can write down symbols and results, but they are not yet able to articulate the relationships among derivative concepts, mathematical models, and solution strategies. This inability reflects a fragmented understanding in which procedures are not integrated with mathematical meaning.

Limitations in mathematical communication also reveal the cumulative nature of the problem-solving process. Difficulties in the interpretation, modelling, and solution-strategy stages accumulate and become apparent during the communication stage. Thus, mathematical communication serves as a final indicator that reflects the overall quality of students' problem-solving processes.

Based on the NVivo analysis, students' written responses, and semi-structured interviews, it can be concluded that students' mathematical communication skills in algebraic function derivatives remain very limited. Students tend to present results and procedures without explaining the conceptual reasons for the steps taken. This finding confirms that mathematical communication cannot develop in isolation; rather, it depends heavily on the success of previous problem-solving stages. Consequently, limitations in mathematical communication constrained students' ability to use mathematics meaningfully beyond symbolic manipulation, which is further discussed in Theme 5.

Theme 5. Meaningful Use of Mathematics in Solving Derivative Problems

The meaningful use of mathematics represents the pinnacle of students' problem-solving quality, where conceptual understanding, modelling, strategy, and communication are coherently integrated. At this stage, students are expected to use the concept of function derivatives not only as a symbolic procedure but also as a tool for understanding and explaining the mathematical phenomena underlying the problem. The analysis shows that only a very small proportion of students were able to use mathematics meaningfully. In contrast, the majority of students still viewed function derivatives as a series of procedural steps detached from context and meaning.

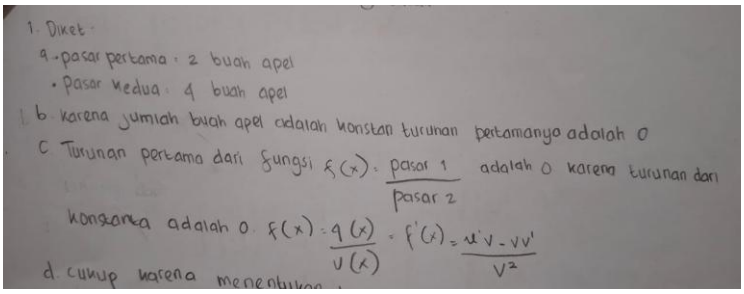


It is known :
Market 1 (1 day before) = 2 apple
Market 2 (3 days later) = 4 apple
for example:
Market 1 = f(x) apple = x
Market 2 = f(y) day = constant
So, f(x) = 2x-1 (u)
f(y) = 4x+3 (v) $f(x) = \frac{2x-1}{4x+3}$
f′(x) = 2 → (u′) $f'(x) = \frac{(2)(4x+3)-(2x-1)(4)}{(4x+3)^2}$
f′(y) = 4 → (v′)

Figure 10. Able to use mathematics meaningfully

Within the broader cognitive cascade framework presented in Figure 1, students' meaningful use of mathematics is more specifically represented by the contrasting responses shown in Figures 10 and Figure 11. The findings indicate that many students had difficulty integrating problem interpretation, mathematical modelling, strategy selection, and mathematical communication into coherent solution processes. In many cases, responses remained dominated by symbolic procedures, with no clear connections among the stages of problem-solving. Only a small proportion of students demonstrated the ability to apply mathematical concepts consistently across interconnected stages of problem-solving.

These difficulties became evident when students were unable to relate derivative calculations to broader mathematical meaning or to the contextual demands of the problems. As a result, students' use of mathematics generally remained procedural and did not develop into coherent conceptual understanding. This pattern reflects the cumulative impact of difficulties encountered across previous stages of problem-solving, particularly in interpretation, modelling, strategy selection, and mathematical communication.



It is known : Market 1 = 2 apple
Market 2 = 4 apple

- because the number of apples is constant, its derivative is 0
- the first derivative of the function $F'(x) = \frac{market\ 1}{market\ 2}$ is 0. Because derivative of constant is 0

Figure 11. Unable to use mathematics meaningfully

Analysis of students' written answers in Figure 10 shows that students who used mathematics meaningfully not only presented derivative results but also related them to the problem's purpose. In this group, derivatives were understood as representations of changes or properties of functions, so that the final results had clear mathematical interpretations.

Conversely in Figure 11, students who were unable to use mathematics meaningfully tended to stop at presenting symbolic results without further explanation. Students' answers generally ended with derivative forms or numerical values, without relating the results to the question's context. This pattern shows that mathematics was treated as an activity of symbol manipulation rather than as a means of building understanding.

The interview results clarify how students interpret derivatives in the problem-solving process. Students who used mathematics meaningfully demonstrated an awareness of the conceptual role of derivatives.

Researcher	:	*"What do you understand about the meaning of the derivative result after differentiating a function?"*
Student ESF	:	*"After deriving it, I can understand how the function changes. So the derivative is not just a number, but can explain the nature of the function."*

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This statement shows that the student had integrated the differentiation procedure with the conceptual meaning of derivatives as a representation of change. Conversely, students who were unable to use mathematics meaningfully reported not understanding the meaning of the results they obtained.

Researcher	:	*"What do you understand about the meaning of the derivative result after differentiating a function?"*
Student TNL	:	*"Usually, after getting the derivative result, I don't know what it means. The important thing is that it has been differentiated according to the formula."*

Some students also stated that they were not accustomed to being asked to interpret mathematical results in their learning.

Researcher	:	*"Are you usually asked to interpret what the derivative result means?"*
Student RKA	:	*"We are rarely asked to explain what the results mean. Usually, it is enough to just give the answer."*

These interviews show that limitations in the meaningful use of mathematics stem not only from students' understanding but also from learning habits that emphasise procedures over the interpretation of results.

Conceptually, students' inability to use mathematics meaningfully reflects the dominance of a procedural orientation over a conceptual one in learning about function derivatives. Students can perform symbolic manipulations, but they do not relate the results to the concepts of change, gradient, or function properties that constitute the essence of derivatives.

The meaningful use of mathematics requires integrating all stages of problem-solving, namely accurate interpretation, relevant modelling, planned strategies, and reflective communication (Fontana & Groenwald, 2023). When one of these stages is weak, the ability to use mathematics meaningfully is hampered. Therefore, failure in Theme 5 is an accumulation of difficulties that arose in the previous themes.

Based on the NVivo analysis, students' written responses, and semi-structured interviews, it can be concluded that the meaningful use of mathematics in the context of algebraic function derivatives has not yet developed optimally in most students. Mathematics is perceived more as a set of procedures that must be completed than as a tool for understanding and explaining mathematical phenomena.

From an APOS perspective, students' reliance on mechanical solution strategies reflects the absence of schema-level integration in their mathematical thinking. Although some students were able to calculate derivatives procedurally, they were unable to connect problem interpretation, mathematical modelling, differentiation procedures, and result interpretation into a coherent conceptual structure. As a result, their reasoning remained fragmented and dominated by symbolic manipulation.

In addition, many students had not yet achieved an object-level understanding of derivatives. Derivatives were viewed primarily as computational procedures rather than as conceptual representations of rates of change in specific contexts. Consequently, students experienced difficulties explaining the meaning of their solutions, justifying strategies, and interpreting derivative results conceptually. Within the APOS framework, these findings indicate fragmented schema construction, where mathematical knowledge remains disconnected rather than integrated into a flexible cognitive structure.

These findings confirm that improving mathematical problem-solving skills cannot be achieved solely by strengthening procedural skills; rather, it requires learning that explicitly emphasises conceptual understanding, the integration of representations, and reflection on mathematical results. Thus, Theme 5 serves as an analytical culmination that summarises the interrelationships among all stages of students' problem solving in the subject of function derivatives.

Discussions

The findings suggest that students' difficulties in solving derivative problems are not best understood as isolated procedural errors, but as a cumulative cognitive breakdown across interconnected stages of mathematical activity. Rather than emerging independently, students' errors formed a hierarchical cascade in which early interpretive and representational difficulties constrained later strategic, communicative, and conceptual performance.

This pattern indicates that students' difficulties with derivatives are fundamentally structural rather than merely procedural. The central problem lies not in students' inability to execute differentiation rules, but in their inability to coordinate interpretation, representation, and conceptual meaning within a coherent problem-solving structure. In this sense, mathematical modelling emerges not simply as one stage among others, but as the principal cognitive hinge upon which subsequent success or failure depends.

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The results of the study indicate that students' problem-solving process is gradual and hierarchical, beginning with problem interpretation, mathematical modelling, selection of solution strategies, mathematical communication, and meaningful use of mathematics. These findings reinforce the theoretical view that problem-solving is not merely a procedural activity but a complex cognitive process involving the integration of conceptual understanding, representation, and reflection.

In Theme 1, students' initial interpretation of function-derivative problems served as the cognitive foundation for the quality of the subsequent stages. Students who failed to identify the problem's meaning from the outset tended to experience a chain of difficulties in modelling, strategy selection, and communication.

These findings are in line with the problem-solving theory of Smith & Nigro, (2023), which emphasises the importance of the problem-understanding stage as a prerequisite for planning and implementing solutions. However, this study expands on this understanding by showing that, in the context of function derivatives, failure to interpret problems is not only related to understanding the language of the question, but also to students' conceptual understanding of the meaning of derivatives themselves.

The findings in Theme 2 indicate that mathematical modelling is the most dominant obstacle in students' problem-solving. Although some students can identify the initial information, the majority struggle in transform the problem context into a relevant mathematical model. This indicates a gap between the ability to understand problems verbally and the ability to represent them symbolically.

Theoretically, mathematical modelling requires the ability to move between representations and construct meaningful relationships between variables (Sandoval Jiménez & Sierra, 2025). The findings of this study indicate that students tend to view functions as symbolic objects to be manipulated rather than as mathematical representations of problem situations. This gap explains why mastery of derivative procedures does not automatically result in good problem-solving skills. Thus, this study confirms that weaknesses in mathematical modelling are not merely technical problems, but conceptual problems rooted in how students interpret functions and derivatives.

In Theme 3, students' problem-solving strategies were dominated by procedural approaches that were not always well planned. Students tended to choose strategies based on their memory of examples or formulas they had learned, rather than on an analysis of the problem structure. These findings support the view that effective problem-solving strategies depend on the quality of planning and understanding of the problem, not merely on mastery of algorithms.

This study shows that failure at the mathematical modelling stage directly limits students' ability to devise appropriate strategies. In other words, solution strategies reflect the quality of students' conceptual and representational understanding. This finding enriches the theoretical discourse on problem-solving strategies by emphasising that strategies are not a stand-alone stage, but rather a product of the success of the previous stages.

The findings in Theme 4 indicate that students' mathematical communication skills remain very limited. Students are generally able to write down results or procedural steps, but they have difficulty explaining the conceptual reasons behind these steps. Theoretically, mathematical communication is seen as a means of externalising the thinking process and reflecting conceptual understanding (Al-Hanifah et al., 2023).

The results of this study indicate that weak mathematical communication is cumulative in nature; that is, it is an accumulation of difficulties in the stages of interpretation, modelling, and problem-solving strategies. Thus, mathematical communication cannot be understood as a separate skill, but rather as an indicator of the overall quality of students' problem-solving processes. This finding makes a conceptual contribution by positioning mathematical communication as a reflection of the integration of cognitive processes, rather than merely as a verbal or mathematical writing skill.

In Theme 5, only a very small proportion of students were able to use mathematics meaningfully in solving derivative problems. Students in this group were able to relate the derivative results to mathematical meanings, such as changes or properties of functions, while most students stopped at symbolic manipulation without interpretation.

This finding confirms that meaningful use of mathematics requires integrating all stages of problem-solving. When one stage is weak, the ability to interpret mathematical results is hampered. Theoretically, this reinforces the view of Sandoval Jiménez & Sierra (2025) that conceptual understanding does not emerge instantly but is built through a coherent, reflective problem-solving process. This study shows that, in function-derivative materials, a dominant procedural orientation is the main obstacle to the development of meaningful understanding.

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Compared to previous studies that generally focus on procedural errors or students' success rates in solving derivative problems, this study offers a comprehensive procedural perspective. The novelty of this study lies in its integrated mapping of students' problem-solving processes, from interpretation to the meaningful use of mathematics, based on NVivo thematic analysis that combines written test data and interviews.

This study also fills a theoretical gap by showing that students' main difficulties with derivative material do not lie solely in mastering differentiation rules, but also in their interpretation of functions, derivatives, and the relationship between problem-solving stages. Thus, this study contributes to the development of mathematical problem-solving theory by emphasising the role of modelling and conceptual meaning as key factors in successful calculus learning at the senior secondary level.

Implication of Research

Theoretically, this study reinforces the view that mathematical problem-solving is a hierarchical, cumulative cognitive process, in which success at each stage depends heavily on the quality of the previous stage. These findings imply that further research should examine problem-solving not in a piecemeal manner, but as a series of interrelated processes, especially in abstract mathematical subjects such as calculus.

For future researchers, the results of this study offer opportunities to develop more in-depth studies of learning interventions that specifically target critical stages, such as problem interpretation and mathematical modelling. Further research could also broaden the context by involving different levels of education, other mathematical materials, or diverse methodological approaches, such as experimental designs or classroom action research, to test the effectiveness of problem-solving-based learning strategies more broadly.

From a learning practice perspective, the findings of this study indicate that an emphasis on mathematics learning that focuses too heavily on mastering derivative procedures is insufficient to develop students' comprehensive problem-solving abilities. Teachers need to design learning activities that explicitly train students to interpret problems, construct mathematical models, and relate calculation results to the conceptual meaning of function derivatives.

Another practical implication is the need to habituate mathematical communication and reflection activities in learning, such as asking students to explain the reasons for their choice of strategy or to interpret the meaning of derivative results in the context of the problem. Thus, students will not only be procedurally skilled, but also able to use mathematics meaningfully.

For policymakers and curriculum developers, the results of this study underscore the importance of aligning curriculum objectives with classroom instruction. Curricula that emphasise higher-order thinking skills and problem-solving need to be accompanied by learning and assessment guidelines that encourage conceptual understanding rather than merely the achievement of final results. Mathematics learning assessments should be designed to comprehensively evaluate students' thinking processes, including their abilities in interpretation, modelling, and mathematical communication.

Limitation

Although this study provides meaningful findings, several limitations need to be considered when interpreting the results. First, this study was conducted in a single school context and with a single group of students; therefore, the findings cannot be generalised to a wider population. The characteristics of students, learning environments, and learning practices in other schools may produce different problem-solving patterns.

Second, the study focused on the derivatives of algebraic functions, so the findings may not necessarily apply to other mathematical materials with different conceptual and representational characteristics. These limitations open up opportunities for further research to examine the problem-solving process in other mathematical topics, such as integrals or function limits.

Third, the research data were obtained through written tests and semi-structured interviews, which relied heavily on students' ability to express their thoughts in writing and verbally. It is possible that these two instruments did not fully reveal some students' thought processes.

Fourth, this study used a qualitative approach with a case study design, so the results emphasise depth of understanding of the process rather than breadth of data coverage. Although this approach is appropriate for the objectives of the study, these limitations may serve as a basis for future studies to use different methodological designs to complement the findings.

These limitations do not diminish the significance of the research findings but rather clarify the scope and context of their interpretation. By understanding these

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limitations, readers and future researchers can use the findings of this study more proportionally and as a basis for further study.

Conclusion

This study concludes that students' problem-solving abilities in algebraic function derivatives involve hierarchical and integrated cognitive processes, in which the successful use of mathematics is largely determined by the quality of problem interpretation, mathematical modelling, selection of solution strategies, and mathematical communication. The findings show that although some students can identify problem elements and apply derivative procedures, the majority experience difficulties at the mathematical modelling stage, which cascades into their solution strategies, their ability to explain the process, and their interpretation of results. These difficulties stem from the dominance of a procedural orientation and fragmented conceptual understanding of functions and derivatives, so mathematics is perceived as symbolic manipulation rather than as a representation of changes and relationships between variables. Thus, this study confirms that improving problem-solving skills in function derivatives cannot be achieved solely by strengthening formula mastery, but requires learning that systematically emphasises conceptual understanding, mathematical model construction, strategy planning, and reflection on the meaning of mathematical results. Future research may build on this model by examining how instructional interventions can reduce cognitive fragmentation and support more integrated mathematical problem solving in calculus learning.

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Additional Information : -

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