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Mathematical Critical Thinking Skills of Students in Solving Set Theory Problems

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ABSTRACT

Mathematical critical thinking is a fundamental competency that students must develop to solve complex problems effectively; however, empirical evidence consistently indicates that this ability remains underdeveloped among junior high school students in Indonesia, particularly in the domain of set theory. This study aims to describe and analyze the mathematical critical thinking skills of seventh-grade students at a public junior high school located in a rural district of West Kalimantan, Indonesia, in solving set theory problems, based on four Facione indicators: interpretation, analysis, evaluation, and inference. A descriptive qualitative design was employed, involving six students selected through purposive sampling from a class of 29 students in the 2025/2026 academic year, categorized into high, medium, and low ability groups. Data were collected through a written essay test and semi-structured interviews, then analyzed using an interactive model comprising data reduction, data display, and conclusion drawing. Results show that high-ability students fulfilled all four critical thinking indicators optimally but demonstrated a consistent limitation in verbal mathematical justification. Medium-ability students performed adequately on interpretation and analysis but encountered difficulties in evaluation and systematic inference. Low-ability students failed to meet most indicators, with the most pronounced deficits in evaluation and inference. This study contributes to the field by investigating the distribution of critical thinking profiles within a unified classroom environment. Crucially, it highlights that verbal mathematical justification represents a fundamental developmental barrier that transcends procedural competence, offering specific implications for the refinement of mathematics instruction.
Keywords: Critical Thinking, Evaluation, Inference, Interpretation, Set Theory.

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Introduction

Mathematics education holds a pivotal role in fostering students' higher-order thinking skills, especially during an era requiring strong analytical, logical, and problem-solving abilities (Lestari et al., 2025, 2026; Uripno & Nurfalah, 2025). Mathematical critical thinking is a key skill in this regard, empowering students to interpret data precisely, develop logical arguments, scrutinize reasoning steps, and arrive at justified conclusions (Aizikovitsh-Udi & Cheng, 2015; Zhu, 2023). Although its importance garners broad consensus, empirical research persistently reveals that students' mathematical critical thinking skills remain critically underdeveloped.

Data from the Program for International Student Assessment indicate that Indonesia ranked 68th out of 81 participating countries in mathematics, achieving a mean score of 366 (Aisyah & Juandi, 2022). This score is substantially below the OECD average of 472 and signifies persistent difficulties in students' capacity for complex mathematical reasoning (Ketaren et al., 2022). This circumstance emphasizes the immediate necessity to explore and mitigate the barriers obstructing critical thinking development within mathematics classrooms (Alhowail & Albaqami, 2024). In particular, the shift from arithmetic operations to abstract logical structures represents a significant cognitive transition that is recognized as a global instructional challenge (Sfard, 1991; Tall, 2013). Consequently, set theory serves as a critical focal point for studying the maturation of student reasoning and the move toward formal mathematical thinking (Bagni, 2006; Vinner, 2002). This universality underscores that addressing critical thinking in core mathematical subjects transcends local Indonesian educational issues, emerging instead as a global imperative that warrants dedicated scholarly attention in alignment with international competency standards, especially regarding foundational subjects like set theory that demand the concurrent application of logical reasoning and abstract conceptualization (Jamil et al., 2024).

Various researchers have conceptualized mathematical critical thinking, and Facione's framework ranks among the most commonly adopted in educational studies (Facione & Facione, 1993). Facione outlined six primary critical thinking skills, namely interpretation, analysis, evaluation, inference, explanation, and self-regulation, with the first four most often applied to assess students' mathematical reasoning (Li et al., 2022). A number of studies have explored mathematical critical thinking skills among junior high school students across different topics and settings (Yohannes et al., 2021). For example, investigations into problem-based learning with eighth-grade students identified major weaknesses in evaluation and inference, as students frequently neglected to validate their solutions' accuracy (Theabthueng et al., 2022).

Similarly, research indicates that junior high school students face substantial challenges with the analysis indicator of critical thinking, especially in problems that demand multi-step logical reasoning (Azmi et al., 2022; Basri et al., 2019). Investigations into critical thinking within algebra contexts reveal that differences across ability levels yield varied performance profiles, where low-ability students regularly struggle to build coherent inferential arguments (Chimoni & Pitta-Pantazi, 2015). Building upon such research, a meta-analysis of problem-based learning models demonstrated substantial gains in mathematical critical thinking skills among junior high school students relative to traditional methods, emphasizing the pivotal role of pedagogical strategies (Syahlan & Simamora, 2022). Recent qualitative studies on critical thinking in geometry contexts have identified limitations in verbal justification and mathematical communication even among high-ability students (Firdaus et al., 2024).

Although prior research has offered valuable contributions, several gaps continue to exist in the current literature (Nguyen et al., 2025). The majority of previous studies have concentrated on algebraic or geometric topics, which has left set theory as a comparatively underexplored area for critical thinking analysis (Khusna et al., 2023). The cognitive demands of set theory differ fundamentally from those of algebra or geometry, as it uniquely requires the simultaneous application of formal logical notation, abstract classification, and relational reasoning through operations such as union, intersection, and complement within a single problem (Sezgin et al., 2025). This stands in contrast to algebraic thinking, which centers on symbol manipulation and pattern recognition, and geometric thinking, which relies primarily on spatial reasoning and proof construction (Moukhliss et al., 2022). Such a distinctive cognitive profile makes set theory a particularly rich yet underexplored domain for studying critical thinking development, warranting dedicated qualitative inquiry. Moreover, numerous studies utilize quantitative or quasi-experimental designs that effectively measure performance differences (Elicer & Neto, 2018). However, they fail to capture the qualitative aspects of students' thinking processes, including the particular reasoning strategies used, the specific points where thinking falters, and the verbal expressions employed in justifying answers.

The existing literature also lacks studies that simultaneously examine critical thinking profiles across all ability levels within a single classroom (Falloon, 2023). This study aims to describe and analyze the mathematical critical thinking skills of seventh-grade students at a public junior high school in rural West Kalimantan, Indonesia, in solving set theory problems, based on four Facione indicators: interpretation, analysis, evaluation, and inference (Facione, 1990, 1995, 2011). This study aims to generate findings that are both theoretically grounded and practically actionable for mathematics educators seeking to design more targeted and effective instructional strategies. Crucially, it highlights that verbal mathematical justification represents a fundamental developmental barrier that transcends procedural competence, offering specific implications for the refinement of mathematics instruction.

Methods

Research Design

This study employed a descriptive qualitative design to describe and interpret students' mathematical critical thinking as it occurs naturally (Sachdeva & Eggen, 2021). Written test results were combined with semi-structured interviews to capture both the products and processes of students' thinking across ability levels.

Participants or Data Sources

The research involved 29 seventh-grade students from a public junior high school in Landak Regency, West Kalimantan, Indonesia, during the 2025/2026 academic year. Purposive sampling was employed to select six students based on their mathematical ability levels, which were classified into high, medium, and low categories (Damayanti & Sumardi, 2018). Prior to selection, all 29 students completed a 12-item questionnaire on mathematical critical thinking, based on Facione's four indicators and using a Likert scale. Importantly, this tool assessed dispositions and self-perceptions rather than direct cognitive performance, aligning with Facione & Facione's original conceptualization of critical thinking dispositions. Students with total scores above 50 were categorized as high ability, those scoring 30-49 as medium ability, and those below 30 as low ability; the maximum point was 60 (Facione & Facione, 1993; Li et al., 2022). Informed consent was secured from the school principal, teachers, and parents prior to commencing data collection.

Research Instruments

Two instruments were used: (1) a written essay test comprising five open-ended items on set theory (set notation, operations, and Venn diagrams), designed to assess the four Facione critical thinking indicators and scored on a 0–2 scale per indicator (maximum 8 points). The scoring criteria were operationalized as follows: a score of 2 was awarded when the student's response fully met the indicator requirements, including correct mathematical notation, complete and logically sequenced reasoning steps, and an appropriate contextual conclusion; a score of 1 was awarded for a partially correct response that demonstrated some understanding of the indicator but contained mathematical errors, reasoning omissions, or an incomplete conclusion; and a score of 0 was awarded for a response that was entirely absent, wholly incorrect, or showed no meaningful engagement with the indicator; and (2) a semi-structured interview guide aimed at exploring students' reasoning processes and difficulties. Expert validation involved two mathematics education specialists who assessed content validity—confirming that each item adequately represented the targeted Facione indicator and aligned with the seventh-grade set theory curriculum—and face validity, confirming item clarity and linguistic appropriateness for the target age group. Following expert review and revision, a small-scale pilot administration was conducted with five students from a comparable class (not included in the main study) to evaluate item readability and generate preliminary difficulty indices; minor wording adjustments were made based on pilot feedback prior to the final administration (Facione, 1990, 1995).

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Research Procedures and Data Collection

The study proceeded through three phases: (1) preparation, including instrument development and validation; (2) data collection, comprising questionnaire administration to all students, selection of six subjects, written test administration, and individual semi-structured interviews (20–30 minutes each, audio-recorded and transcribed); and (3) data analysis and conclusion drawing. Field notes were also recorded to support data interpretation.

Data Analysis Techniques

Data were analyzed using Miles et al.'s (2014) interactive model, comprising data reduction (coding responses according to Facione indicators), data display (organizing into matrices and narrative summaries), and conclusion drawing/verification (Cohen et al., 2007; Siswono, 2019). Data from the written test, interviews, and field notes were triangulated to ensure credibility. Analysis was conducted manually, given the small sample size and the depth-oriented nature of the study.

Result and Discussions

Before the written test, researchers administered a critical thinking questionnaire to all 29 students to categorize their ability levels (Facione & Facione, 1993; Li et al., 2022). They selected six participants via purposive sampling, including high, medium, and low categories (two participants for every category). Each participant then completed a five-item essay test on set theory and took part in an individual semi-structured interview (Facione, 1990). Scorers evaluated the responses using Facione's four indicators (Facione, 2011).

High-Category Critical Thinking (ST1 and ST2)

ST1 and ST2 received questionnaire scores of 108 and 106, respectively. They earned perfect scores of 8/8 on the written test while satisfying all four Facione indicators. For Interpretation, both subjects methodically recognized all provided information prior to beginning solutions. ST1 accurately enumerated Set A = {1, 2, 3, ..., 14} and Set B = {2, 4, 6, ..., 14}. ST1 also clearly documented all four contextual elements for Problem 2. ST2 employed an identical organized method, supplemented by the observation that natural numbers commence at 1. Regarding Analysis, both precisely calculated $A \cap B$ = {2, 4, 6, 8, 10, 12, 14} with appropriate set notation. They further methodically parsed the Venn diagram as follows: sports only = 20 - 8 = 12; arts only = 18 - 8 = 10. For Evaluation, both performed every computational step accurately within a logical progression. These steps comprised sports only = 12; arts only = 10; union = 20 + 18 - 8 = 30; neither = 36 - 30 = 6. Their work exhibited complete numerical accuracy. Both subjects indicated they checked their answers prior to submission. Concerning Inference, both formulated the accurate determination that 6 students participated in neither extracurricular activity. This outcome corresponded precisely to the problem's contextual requirements.

Despite achieving perfect procedural scores, both ST1 and ST2 consistently reported limitations in verbal mathematical justification. The following interview excerpt confirms this shared pattern:

Interviewer	: *"Which part of the set theory problems did you find most difficult to explain? Why?"*
ST1	: *"The hardest part was explaining my steps. If it involves calculation, I can do it, but when explaining my reasoning, I sometimes find it difficult."*
ST2	: *"Explaining the reasons for my answers is somewhat difficult. If it is about calculations, I can follow the steps, but expressing it precisely in words is not easy."*

This disparity between computational proficiency and mathematical communication skills corresponds to findings indicating that even high-achieving students struggle with verbal justification and conceptual articulation (Baran & Kabael, 2021; Rahmawati et al., 2025). This pattern also aligns with arguments that written mathematical communication remains one of the most underdeveloped competencies in Indonesian mathematics education (Chasanah et al., 2020). It results from a classroom culture prioritizing procedural correctness over process explanations.

Medium-Category Critical Thinking (SS1 and SS2)

SS1 and SS2 both earned questionnaire scores of 80 and identical total scores of 6/8. They fully satisfied Interpretation and Analysis but displayed evident shortcomings in Evaluation and Inference. For Interpretation, both accurately recognized all key data from the problems, yet interview evidence indicated that their approach depended more on textual cues than true conceptual comprehension (Li et al., 2022). SS1 noted occasional confusion with lengthy problems, whereas SS2 explained spotting numbers in the text and the end question sentence, a superficial method termed text-dependent rather than conceptually oriented interpretation. Regarding Analysis, both correctly identified $A \cap B$ and calculated accurate Venn diagram elements, but they continually voiced uncertainty in positioning values on the diagram. SS1 reported confusion over intersection placement, while SS2 repeatedly questioned the middle value's accuracy, indicative of analytically sound yet conceptually insecure performance.

On Evaluation, both received 1/2. SS1's calculations were numerically correct, but the solution steps were not written systematically, and the final conclusion was absent;

SS2 executed the first three steps correctly but omitted the critical final derivation.. Neither subject regularly verified their work, though for qualitatively different reasons, as confirmed by the following excerpt:

Interviewer	*: "Do you usually check your answers again? Why?"*
SS1	*: "Not really, I only glance at it because I am afraid of running out of time."*
SS2	*: "Rarely. I usually work straight through without checking because I already feel satisfied with my answer."*

This observation indicates that the lack of answer-checking and unawareness of mathematical verification criteria primarily contribute to poor evaluation skills among medium-ability students (Lämsä et al., 2023). Time management and premature satisfaction represent two distinct barriers that necessitate qualitatively different instructional strategies. Regarding Inference, both students scored 1/2, as SS1 delivered a conceptually focused yet terminologically inaccurate justification for Problem 1, whereas SS2 offered no written justification (Napitupulu, 2017). Both indicated that numerical outcomes alone sufficed without a written conclusion, illustrating partial inferential development where accurate results are obtained but conclusions cannot be formally or meaningfully expressed.

Low-Category Critical Thinking (SR1 and SR2)

SR1 obtained a score of 4/8, while SR2 obtained 2/8, which represents the lowest score among all subjects. Both students exhibited deficiencies across all indicators, featuring progressively larger deficits at higher levels of cognitive complexity. Regarding Interpretation, SR1 correctly identified both sets and all four contextual data points for Problem 2, although interview data showed that this identification was mostly mechanical rather than conceptually grounded. In contrast, SR2 admitted to using guessing as a reading strategy and scored 1/2, a situation termed pseudo-interpretation in which students seemingly identify information correctly without truly grasping its mathematical significance. For Analysis, both students recorded the correct intersection value, but lacked formal set notation, supporting derivation steps, and deliberate reasoning demonstration. SR2 employed backward reasoning by directly inserting the value 6 into the summation $12 + 10 + 8 + 6 = 36$ without first deriving $n = 36 - 30 = 6$, which clearly shows that the correct value came from estimation instead of analysis. Additionally, SR2 explicitly admitted to not understanding the conceptual differences between union, intersection, and complement operations, a pattern recognized as the most common conceptual error in students' set theory work.

Interviewer	*: "Do you usually check your answer again? Why?"*
SR1	*: "No. I don't check because I don't know where to start checking."*
SR2	*: "No. I don't re-check because I am not very confident about the answer I wrote."*

On Evaluation, SR1 received a score of 1/2, while SR2 received 0/2. SR1 showed evaluative paralysis because of a lack of metacognitive knowledge about verifying mathematical answers. SR2 showed evaluative paralysis because of low mathematical self-efficacy. Both cases represent distinct yet equally serious forms of evaluative underdevelopment that demand tailored instructional interventions (Yang, 2024). On Inference, both received 0/2. Both subjects wrote "because Set A and Set B are related to each other" as justification for the intersection. This generic and circular reasoning indicates a fundamental inability to articulate precise mathematical relationships or to deduce new information from given premises (Santoso et al., 2021). This suggests a profound deficit in logical reasoning and conceptual application, preventing them from extending their understanding beyond superficial recognition of relationships within set theory (Junarti et al., 2022; Netti et al., 2019). This aligns with findings where students exhibit difficulties in formulating real-world problems and accurately interpreting solutions due to a lack of contextual understanding and precise mathematical formulation (Mirtasari, 2024). Such deficiencies underscore the prevalence of procedural errors stemming from a lack of fundamental knowledge and the inability to correctly apply mathematical rules, rather than solely conceptual misunderstandings (Dorner et al., 2025).

Conclusion

This study examined and evaluated the mathematical critical thinking abilities of seventh-grade students from a rural public junior high school in West Kalimantan, Indonesia, when addressing set theory problems using Facione's four key indicators: interpretation, analysis, evaluation, and inference. High-performing students showed full command of procedures in every critical thinking area, with reliable checking routines and suitable contextual conclusions. However, they consistently struggled with verbalizing mathematical reasoning, highlighting shortcomings in teaching that prioritizes only procedural skills and neglects mathematical communication development. Mid-level students managed interpretation and analysis well but faltered markedly in evaluation and inference. Their challenges arose from irregular verification practices and missing formal conclusion statements, indicating partial metacognitive insight that mere procedural assurance falls short of full critical thinking. Low-performing students displayed widespread weaknesses in all indicators, especially in evaluation and inference. Their work featured rote data spotting, no formal set notation, confusion over set operations, and failure to build or check conclusions, with evaluation stagnation appearing in varied but equally critical forms tied to metacognitive gaps and poor self-efficacy. All ability levels showed a steady drop-off from basic procedural skills to advanced evaluative and communicative ones, with verbal justification as a shared shortfall. This pattern signals that local math teaching must shift from rote procedures to foster evaluation skills and communication, using sociocultural support for verbal reasoning in real classroom interactions.

The study recognizes key constraints. It is a small sample, six deliberately chosen students from one class in a single school, which limits broader applicability.. Results offer a detailed view of critical thinking variations in this setting, not a general picture of Indonesian junior high students. Future studies should explore larger, varied groups, links to self-regulation and self-efficacy, and test interventions targeting evaluation and inference as key hurdles.

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Author's Declaration

Author Contribution	:	Author 1 (Intan Rimawati): Conceptualization, Writing Original Draft, Data Collection, Data Analysis. Author 2 (Siti Suprihatiningsih): Writing Review & Editing, Author 3 (Gusti Uripno): Validation, Academic Supervision Author 4 (Milagros R. Baldemor): Proofread, Academic Supervision
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