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Didactical Situations and Learning Obstacles in Geometric Sequences among High School Students

Wulan Putri Utami[1*], Mohamad Gilar Jatisunda[2], Davron Aslonqulovich Juraev[3], Mohamed Nor Azhari Azman[4]
[1,2]Universitas Majalengka, Indonesia
[3]Uzbekistan Academy of Sciences, Uzbekistan
[4]Universiti Pendidikan Sultan Idris, Malaysia
wputriutami23@gmail.com
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ABSTRACT

This study examines didactic situations by integrating an analysis of students' learning obstacles in mathematics education, particularly on geometric sequences, which remains a persistent source of conceptual difficulty for students. The purpose of this study is to analyze didactic situations based on the Theory of Didactical Situations and to identify the types and emergence of students' learning obstacles within each stage of the learning process. A descriptive qualitative approach was employed involving 36 eleventh-grade students divided into six groups. Data were collected through classroom observations, conceptual tests, and semi-structured interviews and analyzed using data reduction, data display, and conclusion-drawing techniques. The findings show that all stages of the Theory of Didactical Situations—action, formulation, validation, and institutionalization—were implemented systematically, each characterized by distinct patterns of students' activities and interactions with the milieu. Nevertheless, students still encountered ontogenic obstacles during the action phase related to readiness and confidence, epistemological obstacles during the formulation phase in the form of misconceptions and difficulties in identifying multiplicative patterns, and didactical obstacles during the validation phase, characterized by limited participation and weak argumentative skills in discussions. These results indicate that learning obstacles are closely associated with the characteristics of each stage of the didactical situation. This study contributes an integrated analysis linking didactic situations with the emergence of students' learning obstacles at each stage of the TDS through interactions with the milieu. The findings emphasize the importance of adaptive, student-centered instruction to strengthen conceptual understanding and reduce obstacles to learning in mathematics.
Keywords: Didactical Situations, Geometric Sequences, Learning Obstacles, Mathematics Learning.

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Introduction

Mathematics is a fundamental discipline that develops logical, critical, and systematic thinking skills, which are essential for solving real-life problems and advancing scientific knowledge (Stylianides et al., 2024). Mathematics learning should not only emphasize procedural mastery but also be meaningfully designed to provide opportunities for students to actively and reflectively construct their understanding (Rahma & Rahaju, 2020; Verschaffel et al., 2012). However, in practice, mathematics instruction in schools still tends to focus on procedural approaches, which limits students' opportunities to develop deeper conceptual understanding (Prabowo &

Juandi, 2020). This issue is also evident in the topic of geometric sequences, where students often experience learning obstacles. These obstacles include ontogenic, epistemological, and didactic obstacles, which manifest as difficulties in understanding ratios, growth patterns, and relationships among terms, leading to errors in determining the nth term and the sum of sequences (Anjarrani & Kurniasih, 2023; Magfiroh et al., 2024). Therefore, it is necessary to design more meaningful learning that emphasizes student-centered and conceptually oriented instruction to minimize these obstacles and enhance students' understanding of mathematical concepts.

Learning obstacles refer to the difficulties students encounter when they face problems and are unable to appropriately apply their prior knowledge, as well as challenges in understanding, representing, and generalizing mathematical concepts (Fitriana & Maarif, 2022; Sidik et al., 2025). According to Suryadi (2019), learning obstacles can be classified into three main types: ontogenic, epistemological, and didactical obstacles. Ontogenic obstacles are related to students' readiness to learn, including cognitive development, prior knowledge, and affective factors such as motivation and self-confidence. Epistemological obstacles arise from the nature of knowledge itself, where students' understanding is limited to particular contexts, leading to misconceptions or difficulties when concepts are presented in different representations or situations. Meanwhile, didactical obstacles are associated with instructional factors, such as the design of learning activities, teaching strategies, and how teachers present concepts, which may inadvertently limit students' opportunities to construct meaning. These three types of obstacles are interconnected and may occur simultaneously in the learning process, particularly in abstract mathematical topics such as geometric sequences.

To address this issue, the Theory of Didactical Situations (TDS) provides a relevant framework to explain the relationship between learning interactions and the emergence of learning obstacles. From this perspective, mathematical knowledge is constructed through interactions among students, teachers, and the milieu within a didactical situation that enables meaningful learning processes through the stages of action, formulation, validation, and institutionalization (Brousseau, 2002). In this framework, each stage plays a distinct role in supporting students' knowledge construction. The action stage involves students' initial engagement with a problem situation, where they explore possible strategies based on their prior knowledge without direct teacher intervention. The formulation stage allows students to express, communicate, and represent their ideas, typically through discussion and collaborative work. The validation stage emphasizes the process of evaluating and justifying solutions, in which students compare different approaches, present arguments, and assess the correctness of their reasoning. Finally, the institutionalization stage is led by the teacher, who formalizes students' constructed knowledge into established mathematical concepts and connects their findings to formal mathematical principles. The effectiveness of learning within the TDS framework depends on how well these stages are designed and implemented to facilitate meaningful interactions between students and the milieu.

Several previous studies that employ the TDS framework in mathematics education have reported mixed findings. Research by Hortelano & Prudente (2024) indicates that implementing TDS can help students develop a better conceptual understanding through the structured stages of action, formulation, validation, and institutionalization. In addition, Cipta et al. (2026) found that learning activities designed based on the Theory of Didactical Situations (TDS), through the stages of action, formulation, validation, and institutionalization, can support students in constructing mathematical concepts more meaningfully and enhance their engagement in the knowledge construction process. Meanwhile, Hasan (2022) found that the implementation of TDS in classroom practice is still not optimal, as instruction tends to remain procedure-oriented and does not fully follow all the prescribed stages. As a result, students often receive concepts directly without sufficient opportunities for exploration and validation. Thus, these findings suggest that the effectiveness of TDS largely depends on the quality of the didactical situation design implemented in instruction.

Although various studies have examined didactic situations and learning obstacles in mathematics education, research specifically linking them within the context of geometric sequences remains limited. Studies on didactical situations generally focus on the stages of action, formulation, validation, and institutionalization without deeply analyzing how task characteristics and students' interactions with the learning milieu contribute to the emergence of learning obstacles (Tonra et al., 2024). Meanwhile, studies on learning obstacles tend to identify types of students' difficulties without explicitly relating them to the dynamics of each TDS stage in classroom practice (Munawwaroh et al., 2025; Riastuti et al., 2023). This indicates that the relationship between didactical situation design, interactions within the milieu, and learning obstacles in the learning of geometric sequences has not been examined in a contextualized manner within actual classroom practice.

This study offers novelty by providing a more detailed analysis of the relationship between the structure of didactical situations, the characteristics of the milieu, and learning obstacles within the Theory of Didactical Situations framework in the context of geometric sequences at the senior high school level, where the researcher also acts as the teacher who designs and implements the didactical situations in the classroom. In contrast to previous studies that tend only to relate TDS and learning obstacles in a general manner, this study not only describes the stages of action, formulation, validation, and institutionalization, but also directly examines how students' interactions with the milieu designed by the teacher-researcher at each stage give rise to specific types of learning obstacles. Thus, this study provides a more concrete and contextual explanation of the mechanisms underlying the relationship between instructional design and the emergence of learning obstacles in real classroom practice.

Therefore, this study aims to analyze didactical situations in the learning of geometric sequences using the phases of the Theory of Didactical Situations (TDS): action, formulation, validation, and institutionalization. In addition, this study aims to identify students' learning obstacles, including ontogenic, epistemological, and didactical obstacles, and examine how these obstacles emerge at each stage of the learning process.

Methods

Research Design

This study employed a qualitative descriptive case study design based on the Theory of Didactical Situations (TDS) to analyze didactical situations and students' learning obstacles in geometric sequences within a single classroom. In this study, the researcher acted as both teacher and researcher, responsible for designing and implementing didactic situations in the classroom and for observing students' interactions with the milieu during the learning process. The analysis focused on the stages of action, formulation, validation, and institutionalization to examine students' mathematical activities within the didactical situation. In addition, students' interactions with the milieu were analyzed in depth to identify how learning obstacles (ontogenetic, epistemological, and didactical) emerged during the learning process.

Participants or Data Sources

The subjects of this study were 36 eleventh-grade students from a single class. A purposive sampling technique was employed because the study required participants with specific characteristics relevant to the research focus: students who had not yet received formal instruction on the concept of geometric sequences, thereby allowing their knowledge construction process to be observed more authentically. The selection process was carried out in collaboration with the mathematics teacher, based on classroom learning records and the instructional materials covered. The selected students who met the criteria were then divided into six groups of six students to facilitate collaborative learning and group discussions.

Research Instruments

The instruments used in this study included observation sheets, semi-structured interview guidelines, and conceptual tests. All instruments were validated through expert judgment by one mathematics education lecturer and one senior high school mathematics teacher with experience in teaching geometric sequences. The validation process focused on assessing the content validity of the instruments, including their alignment with the research objectives, relevance to the stages of the Theory of Didactical Situations, and clarity of language. Validation was conducted through discussions and written feedback provided by the validators.

Based on the validation results, the instruments were revised in accordance with the experts' suggestions until they were deemed suitable for use in the study. The recapitulation of the instrument validation results is presented in Table 1 to provide an overview of the validators' assessments of each instrument.

Table 1. Summary of Instrument Validation Results

Instrument	Validator	Revised Aspect	Conclusion
Observation Sheet	Mathematics Education Lecturer	Clarity of Indicators	Valid with revision
	Mathematics Teacher	-	Valid without revision
Conceptual Test	Mathematics Education Lecturer	Item variation (construction)	Valid with revision
	Mathematics Teacher	Difficulty level of items	Valid with revision
Semi-structured Interview Guidelines	Mathematics Education Lecturer	-	Valid without revision
	Mathematics Teacher	-	Valid without revision

Based on the summary table of validation results, most of the research instruments were considered valid and appropriate for use. The observation sheet required minor revisions to clarify the indicators, as suggested by the mathematics education lecturer, while the mathematics teacher considered the instrument valid without revision. For the conceptual test, both validators recommended greater item variation and adjustments to the level of difficulty; therefore, the instrument was deemed valid with revisions. Meanwhile, the semi-structured interview guidelines were considered valid and suitable for use without revision by both validators.

Research Procedures

The research procedures were conducted according to the stages of the Theory of Didactical Situations (TDS): action, formulation, validation, and institutionalization. In this study, the researcher served as both the teacher and the instructor responsible for managing the entire classroom learning process. During the learning activities, students were divided into six groups of six to support collaborative learning and group discussions.

In the action stage, students were presented with contextual problems related to geometric sequences to stimulate initial exploration. During the formulation stage, students discussed their ideas and collaboratively constructed initial mathematical models. In the validation stage, students evaluated and compared their solutions with those of other groups to verify their correctness. Finally, in the institutionalization stage, the researcher, who also acted as the teacher, formalized students' findings by introducing formal concepts of geometric sequences. In addition, each of the four stages of the TDS analysis was systematically examined to determine whether learning obstacles emerged at each stage.

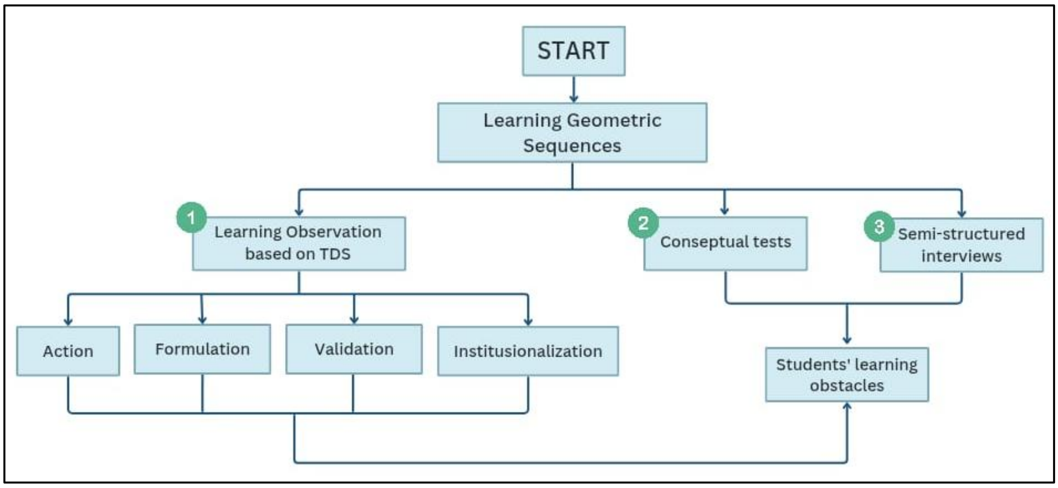


Figure 1. Analysis procedure chart

To support data collection, a conceptual test was administered after the learning process to identify students' understanding and learning obstacles. In addition, semi-structured interviews were conducted with selected students based on their test results to explore in depth the learning obstacles they experienced. These procedures were designed not only to facilitate students' conceptual understanding but also to systematically capture the emergence of learning obstacles at each stage of the learning process. The research procedures are presented in the chart in Figure 1.

Data Collection Techniques

Data in this study were collected using three types of instruments: observation sheets, conceptual tests, and semi-structured interview guidelines. All instruments were systematically developed based on the framework of the Theory of Didactical Situations (TDS), which includes the phases of action, formulation, validation, and institutionalization. This framework ensured alignment between the measured indicators and the analyzed learning stages.

The observation sheet was used to examine students' activities and interactions during the learning process across these TDS phases, including understanding the problem, developing ideas, evaluating solutions, and constructing formal concepts. The conceptual test was administered after the learning process to measure students' understanding and to identify learning obstacles in geometric sequences. The test assessed students' ability to recognize patterns, determine the common ratio and the first term, model contextual problems, and apply formal concepts in problem-solving, all of which align with the TDS phases. In addition, semi-structured interviews were conducted to explore in greater depth the learning obstacles students experienced at each TDS phase. The interview protocol was designed to identify learning obstacles, including ontogenic, epistemological, and didactical obstacles, that emerged throughout the learning process.

To ensure alignment among research objectives, indicators, and data collection techniques, each instrument was complemented by an instrument grid developed based on the TDS stages and the types of learning obstacles examined. The grid for the conceptual test is shown in Table 2, the grid for the conceptual test is presented in Table 3, and Table 4 shows the grid for Interview Guidelines.

Table 2. Grid of Learning Observation

TDS Phase	Indicator	Description of Student Activities	Type of Learning Obstacles
Action	Understanding problems	Students understand the problem and identify the given information	Ontogenic
	Initial exploration	Students begin exploring patterns and relationships.	
Formulation	Developing ideas	Students express ideas to solve the problem	Epistemological
	Constructing models	Students represent the problem in mathematical form.	

TDS Phase	Indicator	Description of Student Activities	Type of Learning Obstacles
Validation	Evaluating Solutions	Students check and evaluate their results	Didactical
	Discussion	Students discuss, compare, and assess solutions.	
Institutionalization	Understanding concepts	Students understand the mathematical concept	Epistemological
	Drawing conclusions	Students formulate conclusions from their solutions.	

Table 3. Grid of Conceptual Test

TDS Phase	Indicator	Description of Measured Ability	Analytical Focus	Type of Learning Obstacles
Action	Understanding problems	Students understand the problem and identify the given information	Accuracy in interpreting information and context	Ontogenic
Formulation	Developing ideas	Students identify patterns or relationships	Ability to recognize patterns and relationships	Epistemological
	Constructing models	Students represent the problem mathematically	Accuracy in modeling the problem	
	Identify concepts	Students determine relevant concepts	Accuracy in using concepts	
Validation	Evaluating solutions	Students check and assess their results	Ability to verify and ensure consistency	Didactical
Institusionalization	Applying formal concepts	Students use formal concepts in solving problems	Consistency in applying formal concepts	Epistemological
	Drawing conclusions	Students formulate conclusions	Clarity and correctness of conclusions	

Table 4. Grid of Interview Guidelines

TDS Phase	Type of Learning Obstacle	Interview Focus	Purpose
Action	Ontogenic	Understanding the problem	To identify students' difficulties in interpreting given information
		Initial response	To examine how students begin solving the problem
Formulation	Epistemological	Understanding patterns and relationships	To identify difficulties in recognizing mathematical relationships
		Model construction	To identify difficulties in representing problems mathematically
Validation	Didactical	Evaluating answers	To identify difficulties in checking and assessing solutions
		Discussion process	To identify obstacles in discussion and idea exchange
Institutionalization	Epistemological	Conceptual meaning	To identify difficulties in understanding concepts holistically
		Generalization	To identify difficulties in drawing conclusions or making generalizations

Data Analysis Techniques

Data were analyzed using a qualitative descriptive approach grounded in the Theory of Didactical Situations (TDS). The analysis was conducted by identifying students' activities at each stage of the didactical situation, namely action, formulation, validation, and institutionalization. At each stage, students' interactions with the milieu were examined to understand how the learning environment responded to students' strategies and responses.

The data analysis process followed the stages of data reduction, data display, and conclusion drawing as proposed by Miles & Huberman (1992). In the data reduction stage, the researcher selected, focused on, and organized data obtained from observations, tests, and interviews by categorizing them according to the TDS stages (action, formulation, validation, and institutionalization) and the types of learning obstacles (ontogenetic, epistemological, and didactical). Data that were not relevant to the study's focus were eliminated, while essential data reflecting students' interactions with the milieu and the emergence of learning obstacles were retained.

In the data display stage, the reduced data were systematically organized into narrative descriptions, tables, and structured interpretations for each TDS stage. This presentation aimed to clarify patterns of students' activities, forms of interaction with the milieu, and types of learning obstacles that emerged at each stage of the learning process. Finally, in the conclusion-drawing stage, the researcher interpreted the displayed data to identify relationships among the stages of didactical situations, the characteristics of students' interactions with the milieu, and the emergence of learning obstacles. Conclusions were drawn iteratively and continuously verified against the data to ensure the consistency and validity of the findings.

Result and Discussions

Result

Analysis of Didactical Situation Based on the Theory of Didactical Situation Phases

In the learning process, the researcher presented a contextual problem related to geometric sequences. The problem given was as follows: *"A cake seller produces 2 boxes of cakes on the first day. On each subsequent day, the number of cake boxes produced doubles from the previous day. Determine the number of cake boxes produced on the 5th day and identify the pattern formed."* To facilitate analysis of the didactic situation, the researcher employed several observation indicators to assess students' interactions, conceptual understanding, and engagement during the learning activities. The observation indicators of the didactical situation are presented in Table 5.

Table 5. Observation Sheet of Didactical Situations

Situation	Observation Indicators	Category	
		Implemented	Not Implemented
Action	The researcher presented problems related to the patterns of geometric sequences.	✓	
	The students read and analyzed the given problems to understand their content.	✓	
	The students began to observe the given number patterns.	✓	
	The students attempted to determine the next term in a sequence.	✓	

Situation	**Observation Indicators**	**Category**	
		Implemented	**Not Implemented**
Formulation	The students began discussing in groups to deepen their understanding of the problem.	✓	
	The students shared their ideas about the sequence patterns within their groups.	✓	
	The students recorded the results of their group discussions on the worksheet.	✓	
	The students explained their problem-solving strategies to their group members.	✓	
	The students collaborated to identify the pattern of the geometric sequence.	✓	
	The researcher facilitated the students in expressing their ideas and opinions.	✓	
Validation	The students reviewed the answers they obtained within their groups.	✓	
	The students compared the results of their discussions with those of other groups.	✓	
	The students provided justifications for their answers.	✓	
	The students responded to or commented on other groups' answers.		✓
	The researcher facilitated a class discussion to evaluate the students’ answers.	✓	
Institutionalization	The researcher connected the students’ discussion results to the concept of geometric sequences.	✓	
	The researcher explained the concept of the ratio in geometric sequences.	✓	

Situation	Observation Indicators	Category	
		Implemented	Not Implemented
	The researcher explained the formula for the n-th term of a geometric sequence.	✓	
	The researcher emphasized the lesson's key conclusions.	✓	
	The students demonstrated understanding of the concepts explained.	✓	

Action Phase

Based on the observation results in Table 1, all indicators in the action phase were optimally achieved, as reflected by the emergence of all observed aspects during the learning process. The researcher presented a contextual problem related to geometric sequences, namely the production of cake boxes that double each day. After the problem was given, students read and comprehended the problem statement, then identified key information and observed the emerging number pattern, such as 2, 4, 8, 16, and so on. Students began to determine whether the pattern continued and engaged in group discussions.

During the process, students used simple language to explain the identified pattern, such as "multiplied by 2 continuously" or "doubles each time." In addition, some students wrote different patterns, such as 2, 4, 6, 8, indicating variations in how they interpreted the information provided in the problem. In group discussions, students compared their answers and communicated their reasoning to other group members.

Formulation Phase

In the formulation phase, all indicators were implemented as students developed their understanding through group discussions by examining different patterns, namely arithmetic-based (2, 4, 6, 8) and geometric-based (2, 4, 8, 16) sequences. The emergence of different responses indicates that students experienced epistemological obstacles in distinguishing between additive and multiplicative structures of patterns. This was influenced by the didactic situation, in which the given task allowed multiple interpretations without explicit guidance toward the concept of geometric sequences.

During the discussion, students compared their results and provided justifications for their chosen patterns. However, some students initially justified the additive pattern based on prior knowledge of arithmetic sequences, indicating that previously constructed schemas still dominated their reasoning. Through interaction within the milieu, including peer discussion and teacher guidance through probing questions, students gradually revised their thinking and reconstructed their understanding toward the correct multiplicative pattern. Students then recorded the agreed results and presented their solution strategies to the class. This process reflects how the milieu functioned as a regulatory system that triggered cognitive conflict and supported conceptual refinement, both of which are closely related to the emergence and resolution of learning obstacles.

Validation Phase

During the validation phase, students reviewed the results of their group discussions by checking the consistency of the identified patterns and the accuracy of their calculations. The similarity of results across groups, which predominantly identified a multiplication-by-2 pattern, indicates that most students had begun to overcome epistemological obstacles encountered in the previous phase in distinguishing between arithmetic and geometric sequences. This shift occurred as a result of cognitive conflict that emerged during the formulation phase, in which interactions within the milieu encouraged students to revise their initial understanding.

Furthermore, students compared their results with those of other groups and justified their answers by explaining that each term is obtained by multiplying the previous term by 2. However, although most groups reached the same conclusion, intergroup interaction remained limited, indicating that the validation process had not fully developed into deep mathematical argumentation. This condition reflects a didactic obstacle: insufficient encouragement for critical discussion between groups limited the refinement of conceptual understanding. Therefore, the validation phase serves not only as a verification stage but also as a process of strengthening and reconstructing knowledge, influenced by the dynamics of the milieu and the persistence of students' learning obstacles.

Institutionalization

During the institutionalization phase, the researcher connected students' discussion results with formal mathematical concepts by emphasizing that the correct pattern is obtained through repeated multiplication of the previous term. This phase was implemented because, in the previous stages, students still exhibited learning obstacles, particularly epistemological difficulties in generalizing geometric sequence patterns and a tendency to apply arithmetic reasoning. Therefore, institutionalization was necessary to redirect students' constructed knowledge toward formally correct mathematical concepts.

The researcher then introduced the concept of a geometric sequence and explained the ratio as the constant multiplier between consecutive terms, which in this case is 2. Subsequently, the general formula of the n-th term was gradually presented by linking it to the pattern previously identified by the students. This process served not only to formalize mathematical knowledge but also to refine students' conceptual understanding and reduce remaining epistemological obstacles. However, the effectiveness of institutionalization was also influenced by students' initial understanding, as some still needed additional practice in applying the formula. Thus, the institutionalization phase functions not merely as the delivery of formal concepts but also as a reconstruction of students' knowledge, influenced by the dynamics of the milieu and the persistence of learning obstacles.

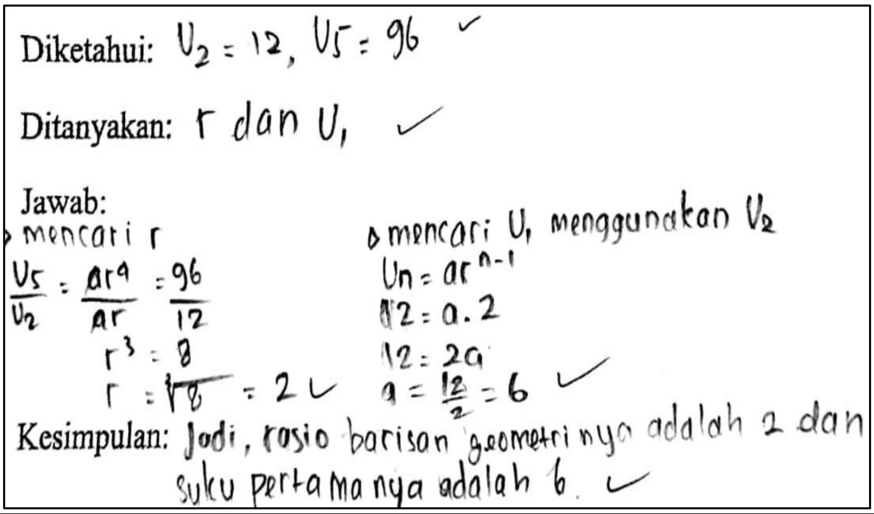
Analysis of Students' Learning Obstacles

After the learning activities were completed, students were given a worksheet to be worked on in groups. Each group was asked to solve four problems on geometric sequences as an evaluation of their conceptual understanding of the material learned, as well as to identify students' learning difficulties. The problems given to the students are presented in Table 6.

Table 6. Problems in the Student Worksheet

Question Number	Problems	Questions
1	There is a geometric sequence in which the second term is 12, and the fifth term is 96.	Determine the common ratio and the first term.
2	The number of prospective Hajj pilgrims in a province in the first year is 1,000, and it doubles each year thereafter.	The number of prospective Hajj pilgrims in the 5th year is?
3	A bacterium reproduces by quadrupling every 10 minutes. After 20 minutes, the number of bacteria becomes 64.	What is the initial number of bacteria?
4	Nur is a crochet wallet artisan whose monthly production increases according to a geometric sequence. The number of products in the first month is 27 wallets, and in the third month, it is 108.	What is the monthly growth ratio of Nur's crochet wallet production?

In the first problem, students were asked to determine the common ratio and the first term of a geometric sequence based on the second and fifth terms. The analysis showed that most groups accurately reconstructed the relationships among the terms. They did not merely apply formulas but also constructed logical relationships among sequence elements to determine the ratio and the first term. This indicates that students had a solid understanding of the fundamental concepts of geometric sequences. To clarify the students' solution process, Group 5's work is presented in Figure 2.


Diketahui: $U_2 = 12$, $U_5 = 96$

Ditanyakan: r dan U_1

Jawab:

- mencari r

$\frac{U_5}{U_2} = \frac{ar^4}{ar} = \frac{96}{12}$

$r^3 = 8$

$r = \sqrt[3]{8} = 2$

- mencari U_1 menggunakan U_2

$U_n = ar^{n-1}$

$12 = a.2$

$12 = 2a$

$a = \frac{12}{2} = 6$

Kesimpulan: Jadi, rasio barisan geometrinya adalah 2 dan suku pertamanya adalah 6.

Translate in English:

Known: $U_2 = 12$, $U_5 = 96$

Asked: r and U_1

Answer :

- Determining r

$$\frac{U_5}{U_2} = \frac{ar^4}{ar} = \frac{96}{12}$$
$$r^3 = 8$$
$$r = \sqrt[3]{8} = 2$$

- Determining U_1 using U_2

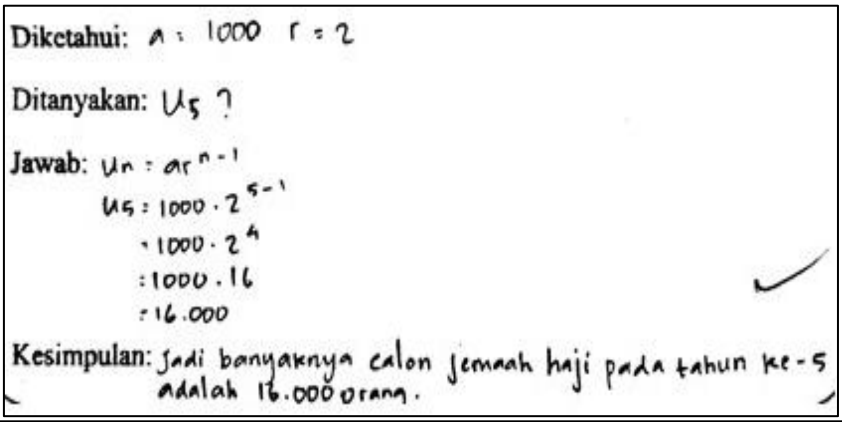
$$U_n = ar^{n-1}$$
$$12 = a.2$$
$$12 = 2a$$
$$a = \frac{12}{2} = 6$$

Conclusion: Therefore, the common ratio of the geometric sequence is 2 and the first term is 6.

Figure 2. Group worksheet for question 1

Based on Figure 2, it can be seen that students were able to determine the value of the ratio from $\sqrt[3]{8}$ As 2, this indicates that students still retained their understanding of exponentiation concepts. In addition, students were able to determine a particular term by substituting the known elements into the geometric sequence formula.

In contrast to the first problem, the second problem required students to model a contextual situation involving the number of Hajj candidates in the fifth year. In this context, students were not only working with numerical data but also translating real-world situations into mathematical representations. All groups solved this problem correctly. An example of Group 4's solution process is presented in Figure 3.


Diketahui: a = 1000 r = 2
Ditanyakan: U5 ?
Jawab: Un = ar n-1
U5 = 1000 . 2 5-1
= 1000 . 2 4
= 1000 . 16
= 16.000
Kesimpulan: Jadi banyaknya calon jemaah haji pada tahun ke-5 adalah 16.000 orang.

Translate in English:
Known: $a = 1000$, $r = 2$
Asked: U_5
Answer:
$U_n = ar^{n-1}$
$U_5 = 1.000 . 2^{5-1}$
$= 1.000 . 2^4$
$= 1.000 . 16$
$= 16.000$
Conclusion: Therefore, the number of prospective Hajj pilgrims in the 5th year is 16.000 people.

Figure 3. Group worksheet for question 2

Figure 3 shows that students were able to determine the first term, the common ratio, and the value of a specific term by constructing a mathematical model from the word problem. This indicates that students accurately transformed verbal information into a geometric sequence model. In addition, students were able to provide conclusions consistent with the problem context, suggesting that their mathematical representation skills are beginning to develop and that their computational results can be connected to the given context.

In the third problem, which involved bacterial growth, students' ability to connect contextual information to the concept of geometric sequences varied. Although all groups correctly determined the common ratio, only some students were able to associate the value 64 with its corresponding term position in the sequence. This suggests that the main difficulty lies not in procedural computation but in connecting the temporal context with the structure of sequence terms. The differences in students' work are shown in Figures 4 and 5.

Department of Mathematics Education, Universitas Muhammadiyah Purwokerto, Purwokerto, Indonesia
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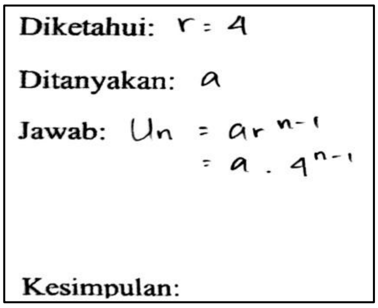
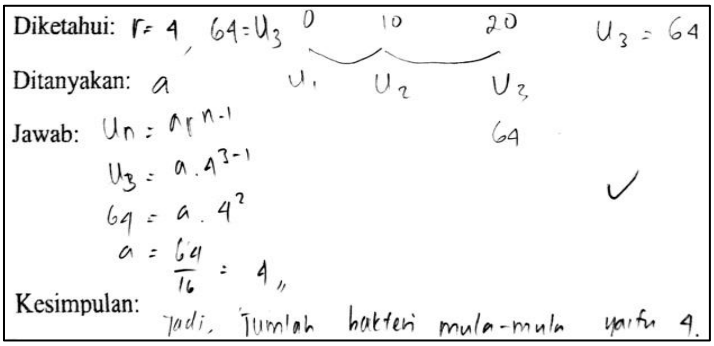


Diketahui: $r = 4$
Ditanyakan: a
Jawab: $U_n = ar^{n-1}$
$= a \cdot 4^{n-1}$
Kesimpulan:

Translate in English:
Known: $r = 4$
Asked: a
Answer:
$U_n = ar^{n-1}$
$\quad = a \,. 4^{n-1}$
Conclusion:

Figure 4. Group worksheet for question 3 with incomplete or incorrect solutions



Diketahui: $r = 4$, $64 = U_3$ 0   10   20 $U_3 = 64$
Ditanyakan: a U_1 U_2 U_3 64
Jawab: $U_n = ar^{n-1}$
$U_3 = a \cdot 4^{3-1}$
$64 = a \cdot 4^2$
$a = \frac{64}{16} = 4$
Kesimpulan: Jadi, Jumlah bakteri mula-mula yaitu 4.

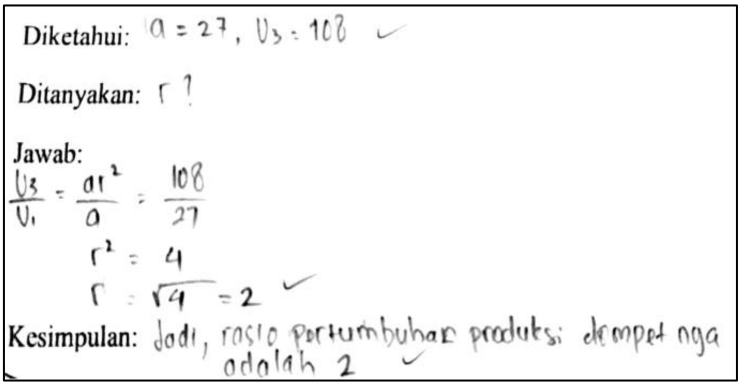
Translate in English:
Known: $r = 4,\ 64 = U_3$
Asked: a
Answer:
$U_n = ar^{n-1}$
$U_3 = a \,. 4^{3-1}$
$64 = a \,. 4^2$
$a = \frac{64}{16}$
$a = 4$
Conclusion: Therefore, the initial number of bacteria is 4.

Figure 5. Group worksheet for question 3 with correctly completed solutions

Based on Figures 4 and 5, there are noticeable differences in students' understanding when solving the problem. In Figure 4, the group was unable to identify that the value 64 corresponds to the third term of the geometric sequence, even though they had correctly determined the ratio. Students were unable to relate time information to the number of terms, resulting in an inaccurate mathematical model. In contrast, in Figure 5, Group 6 correctly integrated information about the ratio and the sequence of time, enabling them to accurately determine the position of the term and obtain the correct answer. This indicates that students' difficulties lie in connecting the problem's context to the concepts of a geometric sequence.

In the fourth problem

, students were asked to determine the common ratio based on the relationship between the first and third terms in a production context. All groups identified that the relationship among the terms followed a geometric sequence, which could therefore be modeled mathematically. In their solution process, students applied the concept of geometric sequences by stating that the third term is obtained by multiplying the first term by the square of the common ratio. Based on this relationship, students constructed a mathematical connection between the first and third terms to determine the value of the ratio. As an illustration, the work of Group 3 is presented in the following figure.


Diketahui: $a = 27$, $U_3 = 108$

Ditanyakan: r ?

Jawab:

$$\frac{U_3}{U_1} = \frac{ar^2}{a} = \frac{108}{27}$$

$$r^2 = 4$$

$$r = \sqrt{4} = 2$$

Kesimpulan: Jadi, rasio pertumbuhan produksi dompet nya adalah 2

Translate in English:
Known: $a = 27,\ U_3 = 108$
Asked: r
Answer:

$$\frac{U_3}{U_1} = \frac{ar^2}{a} = \frac{108}{27}$$

$$r^2 = 4$$

$$r = \sqrt{4} = 2$$

Conclusion: Therefore, the growth ratio of the wallet production is 2.

Figure 6. Group worksheet for question 4

Figure 6 shows that students were able to construct a correct mathematical model and perform systematic calculations to obtain a common ratio of 2. This indicates that students were able to connect the problem context to a mathematical model and had a sound understanding of the concept of ratio in geometric sequences.

To complement the observational and test data, this study also conducted semi-structured interviews with six students, one from each group, to deepen understanding of their conceptual understanding and learning difficulties with geometric sequences. The interview data were analyzed with respect to ontogenic, epistemological, and didactic obstacles.

In terms of ontogenic obstacles, several students experienced confusion at the beginning of the learning process, particularly in understanding the term "doubling," as one student noted: *"At first I was confused because I couldn't really imagine what doubling meant."* In addition, some students lacked confidence in solving problems independently, for example, *"I am not sure about my own answer, so I usually wait for my friends first."* Some students also stated that they still relied heavily on teacher-provided examples, such as *"If it hasn't been demonstrated, I usually don't really understand yet."*

Regarding epistemological obstacles, some students still had misconceptions about geometric sequences, particularly the meaning of "doubling," as illustrated by the response, *"I thought doubling means adding 2, so the pattern is 2, 4, 6, 8."* Students also reported difficulties in relating the concept to word problems, for example, *"Word problems are quite difficult; I get confused about which term it refers to."* However, some students already demonstrated a correct understanding of the basic concept, such as *"A geometric sequence is when you keep multiplying by the same number."*

Regarding didactic obstacles, several students stated that group discussions helped their understanding; for example, *"Group discussions really help because we can ask each other when we don't understand."* However, some students were still less active in expressing their opinions, such as *"I rarely speak in front of others because I am afraid of being wrong."* In addition, some students found it difficult when faced with problems that differed from the examples provided; for instance, *"If the problem is different from the example, I get confused in solving it."* Students also mentioned that the teacher's explanation supported their understanding: "When the formula was explained, I understood better why the pattern works that way."

Discussions

The findings of this study indicate that the learning process based on the Theory of Didactical Situations (TDS) is effective in facilitating students' conceptual understanding of geometric sequences. In each phase, namely action, formulation, validation, and institutionalization, students gradually constructed their understanding through interaction with contextual problems, collaborative discussion, and the formalization of mathematical concepts. This study analyzes didactical situations and students' learning obstacles using TDS, emphasizing the interrelationship between the emergence and development of learning obstacles across each learning phase, thereby providing a more integrated perspective on the dynamics of students' learning processes. These findings are consistent with previous research indicating that TDS-based learning supports the construction of students' conceptual understanding, and they extend these findings by revealing in more detail the dynamics of learning obstacles across each TDS phase. This aligns with the research objective, which is to analyze didactic situations and students' learning obstacles in the learning process using TDS. However, the effectiveness of each phase was not uniform, indicating that TDS implementation still needs improvement in certain areas.

In the action phase, students began interacting with contextual problems that triggered initial exploration of geometric sequence patterns. The emergence of misconceptions, particularly the interpretation of "doubling" as addition, reflects students' initial cognitive state. This is consistent with the constructivist perspective, which emphasizes that prior knowledge and informal experiences influence problem-solving processes (Radford, 2021). These misconceptions did not arise randomly but were influenced by prior learning experiences that were more dominated by arithmetic sequences, leading students to apply additive strategies to interpret situations that should be multiplicative. Thus, the cognitive conflict that emerged provided an important basis for concept reconstruction, indicating that the selection of contextual problems plays a role in eliciting and guiding the correction of students' misconceptions and must therefore be carefully considered in instructional design.

During the formulation phase, students developed and communicated ideas through group discussions, resulting in meaning negotiation and collaborative refinement of understanding. These findings support social constructivist theory, which emphasizes the role of dialogue in knowledge construction (Alexander, 2020; Mercer, 2019). Through peer interaction, students were able to compare various interpretations of patterns and direct their understanding toward more accurate concepts. This is in line with Tao & Chen (2023), who state that discussion-based learning can deepen understanding through collaborative interaction. Although all groups actively participated in discussions, the quality of interaction varied, particularly in the depth of argumentation and clarity of conceptual explanation. This condition indicates that active participation in discussion does not automatically ensure deep conceptual understanding, as some interactions remained limited to the exchange of answers without reflective elaboration. Therefore, more structured discussion management is required to ensure that student interactions focus not only on outcomes but also on argumentation, clarification, and deeper exploration of ideas.

In the validation phase, students re-examined their work and compared it with other groups' work. Although most groups obtained similar results, critical interactions such as argumentation and counter-argumentation remained limited. This indicates that students' mathematical argumentation skills have not yet developed optimally, particularly in providing justification and evaluating solutions. This finding is

Department of Mathematics Education, Universitas Muhammadiyah Purwokerto, Purwokerto, Indonesia
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consistent with studies that argue that argumentation is a crucial component of mathematical thinking, as it supports logical reasoning and conceptual validation (Kurniawan et al., 2023; Nurruzzahra & Maarif, 2024; Salgado et al., 2024). This limitation shows that similar results do not automatically reflect similar understanding, because students tend to focus on final answers without exploring the reasoning behind the procedures used. This suggests that the validation process has not fully functioned as a space for testing conceptual understanding through in-depth argumentation. Therefore, more structured learning activities are needed to encourage students to present reasons, defend their ideas, and critically evaluate solutions through argumentative discussion.

In the institutionalization phase, the teacher formalizes students' informal knowledge by introducing the concepts of geometric sequences, ratios, and the general term formula. This process bridges students' experiential understanding with formal mathematical structures in accordance with the TDS framework (Brousseau, 2002; Suryadi, 2019). The teacher's intervention serves as a process of validating and generalizing students' constructed knowledge, making the concepts more systematic and mathematical. However, this formalization process must be carried out carefully to avoid reducing students' opportunities to construct meaning independently. If teacher intervention becomes too dominant, there is a risk that the resulting understanding will be procedural rather than deeply conceptual. This condition calls for a balance between teacher guidance and student autonomy so that institutionalization functions not only as the delivery of formal concepts but also as reinforcement of students' constructed knowledge, which must be carefully designed in instruction.

Based on students' work in solving several problems on geometric sequences, the findings show that most students have understood the basic concept of geometric sequences. The similarity observed among nearly all students is their ability to determine the common ratio and the first term, as well as to use exponential numbers to establish relationships between terms. This indicates that the learning process has resulted in a relatively uniform level of procedural mastery. However, differences begin to emerge at the level of conceptual understanding. Some students can only apply procedures without understanding the mathematical reasoning behind the steps, while others can explain the relationship between procedures and the concept of geometric sequences more comprehensively. This condition indicates that correct problem-solving does not fully reflect conceptual understanding, as it is still influenced by the dominance of procedural activities in the learning process.

This ability shows that students are not merely memorizing formulas, but have understood the structure of geometric sequences and the underlying mathematical concepts. Understanding of exponents and roots is also evident in students' ability to determine the ratio of $\sqrt[3]{8} = 2$, which represents an important aspect in systematically solving geometric sequence problems (Marlina et al., 2024). When comparing students, differences in patterns of understanding can be observed between those oriented toward procedural outcomes and those able to interpret the structural relationships between terms in geometric sequences. These differences indicate that the didactical design process has not yet fully provided equitable learning experiences in connecting numerical representations with conceptual meaning.

In addition to conceptual understanding, students demonstrate good mathematical representation skills. They can construct mathematical models from given information and relate them to the correct solution steps. The similarity observed is that all students can transform verbal representations into symbolic forms, but differences arise when these representations are used to reason more deeply about relationships between terms. This ability is important for translating information from verbal or numerical forms into symbolic representations, thereby facilitating the solution of more complex problems (Afifah et al., 2024; Susmina & Marlina, 2024). With this skill, students were not only able to determine the common ratio and the first term but also to connect numerical patterns with algebraic concepts and the broader structure of sequences, making problem-solving more systematic and logical (Musa et al., 2024). Representation skills also helped students understand the relationships among elements in contextual problems, thereby strengthening conceptual understanding and problem-solving ability (Pramayshela, 2024).

The analysis also reveals variations in students' abilities, particularly when dealing with contextual problems. Some students still experience difficulties in linking contextual information, such as time position or sequence order, with mathematical concepts. As a result, the mathematical models they construct are often inaccurate, even though procedural steps such as determining the ratio are correct. In comparison, students who successfully solve contextual problems are able to integrate situational information with the structure of sequence concepts. In contrast, students who experience difficulties tend to rely solely on computational procedures without establishing connections between elements in the context. In line with the study by Kairudin et al. (2025), which shows that students often struggle to relate real-world contexts to abstract mathematical structures. This finding indicates that procedural understanding alone is not sufficient without strong conceptual and contextual understanding. In other words, students require additional reinforcement to fully understand the relationship between information in contextual problems and the concept of geometric sequences.

On the other hand, students who were able to integrate concepts and procedures could solve the problems accurately and systematically. They did not merely calculate the ratio and terms, but also understood the relationships among terms in geometric sequences and were able to explain the connections between the given elements in the sequence. This indicates that students were not only focused on computational steps but also understood the meaning of each process involved in problem-solving. Thus, the mastery of both concepts and procedures simultaneously indicates that students have developed a more comprehensive understanding of geometric sequences, particularly in connecting patterns, formulas, and computational results. This ability is reflected in the consistency between the mathematical models they construct and the final results obtained, making their solutions more systematic and logical.

In TDS, the analysis of students' worksheets indicates that student success occurs when the action, formulation, and validation phases are implemented optimally, thereby enabling the independent construction of conceptual relationships. Conversely, student difficulties suggest that the institutionalization process has not yet fully succeeded in transforming procedural experiences into stable conceptual understanding. This confirms that the quality of didactical design plays an important role in shaping the depth of students' understanding of geometric sequences.

Based on semi-structured interviews with six students, it was found that learning difficulties in geometric sequences can be classified into three categories: ontogenic, epistemological, and didactic obstacles. This classification is consistent with the learning obstacle theory, which explains that students' difficulties may arise from individual readiness, conceptual understanding, and instructional factors (Suryadi, 2019). However, unlike studies that separate these obstacles, this study shows that they are interconnected and occur simultaneously, indicating that learning difficulties are integrated.

In the ontogenic aspect, some students still experienced limitations in learning readiness and self-confidence, reflected in their confusion in understanding the concept of ratio in geometric sequences and in their lack of confidence in solving problems independently. This finding is consistent with studies showing that self-confidence influences students' independence in solving mathematical problems (Mazana et al., 2018). This condition indicates that affective factors play an important role in students' readiness to construct understanding independently. In addition, there was variation in students' learning independence, with some attempting to solve the problems individually before engaging in group discussion.

In the epistemological aspect, misconceptions were found in understanding the concept of geometric sequences, particularly in interpreting multiplication as addition. This indicates that the concept of ratio has not yet been formally established. This finding is consistent with the study by Kadarisma et al. (2020), which states that misconceptions arise from weak conceptual understanding and differences in students' interpretations. In addition, students also experienced difficulties in relating the concept to contextual problems, especially in determining the position of a term, indicating that the understanding of abstract concepts still needs to be strengthened in mathematics learning (Yusup, 2023). This suggests that the formulation phase has not fully supported the validation phase, indicating the need for tasks that bridge context and formal representation.

Next, in the didactical aspect, most students responded positively to the learning process, particularly through group discussions that helped them understand the material collaboratively. This is in line with Gillies (2019), who found that cooperative learning can promote student interaction through structured discussion, thereby supporting the development of deeper conceptual understanding. However, in this study, not all students were fully engaged, as some still exhibited passive behavior during the discussion. Teacher scaffolding also played an important role in helping students reconstruct their understanding, consistent with findings that scaffolding supports the development of understanding and learning independence (Van De Pol et al., 2010).

Overall, this study indicates that learning based on the Theory of Didactical Situations (TDS) can facilitate students' understanding of geometric sequences. However, the effectiveness of each phase has not been equally achieved. However, this study has limitations related to its context, which is confined to a single class and topic, and to its reliance on interview data and students' written work. Therefore, future research is recommended to examine didactical situations across more diverse contexts and mathematical topics, employ a wider range of data collection techniques, and develop instructional designs that are more adaptable to differences in students' readiness.

Conclusion

This study concludes that implementing the Theory of Didactical Situations (TDS) through the stages of action, formulation, validation, and institutionalization effectively supports students' conceptual understanding of geometric sequences while simultaneously revealing the nature of their learning obstacles. The findings show that well-structured didactic situations enable students to construct knowledge actively, particularly in recognizing patterns, determining ratios, and developing mathematical models. However, the study also identifies that learning obstacles remain present and are systematically linked to each stage of the learning process, including ontogenic obstacles related to students' readiness and confidence, epistemological obstacles in the form of misconceptions and difficulties in interpreting contextual problems, and didactical obstacles associated with limited participation and interaction during learning.

This study contributes to mathematics education by presenting an analysis that connects didactical situations across the phases of the Theory of Didactical Situations (TDS) with the emergence of learning obstacles in the topic of geometric sequences. The findings show that in the action phase, students begin to explore patterns through problem situations; in the formulation phase, ideas are exchanged and negotiated through group discussions; in the validation phase, students compare and evaluate solutions, although their argumentation remains limited; and in the institutionalization phase, the teacher formalizes the concepts constructed by students. The learning obstacles identified include ontogenic obstacles, such as students' limited initial readiness and low self-confidence in understanding the concept of ratio; epistemological obstacles, such as misconceptions in interpreting multiplication as addition, difficulties in determining the position of terms, and challenges in connecting contextual situations with the concept of geometric sequences; and didactical obstacles, such as limited student participation in discussions and underdeveloped argumentation during the validation process. In addition, the study shows that students' engagement in discussion and the teacher's scaffolding influence the learning process, although not uniformly across all students. In practice, these findings highlight the importance of designing adaptive, student-centered instruction that supports students' active engagement, conceptual understanding, and participation in mathematical discussions.

Nevertheless, this study has limitations related to its context: it is confined to a single class and a single topic and relies on data from interviews and students' written work. Therefore, future research is recommended to examine didactical situations across more diverse contexts and mathematical topics, employ a wider range of data collection techniques, and develop instructional designs that are more adaptive to differences in students' readiness. Overall, this study provides meaningful insights for educators, researchers, and practitioners in designing more effective, conceptually rich, and responsive mathematics learning.

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Author's Declaration

Author Contribution	:	Author 1: Conceptualization, Methodology, Investigation, Data Curation, Formal Analysis, Visualization, Writing - Original Draft Author 2: Conceptualization, Methodology, Supervision, Validation, Writing - Review & Editing Author 3: Validation, Writing - Review & Editing Author 4: Validation, Writing - Review & Editing
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